COMPTES RENDUS

(DOKLADY)

DE L'ACADÉMIE DES SCIENCES DE L'URSS

NOUVELLE SÉRIE

1939

VOLUME XXIII

№ 9

ÉDITION DE L'ACADÉMIE DES SCIENCES DE L'URSS
MOSCOU

AVIS AUX AUTEURS

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COMPTES RENDUS

(DOKLADY)

DE L'ACADÉMIE DES SCIENCES DE L'URSS

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NOUVELLE SÉRIE

7-me année

Paraissant tous les dix jours

1939

VOLUME XXIII, № 9

TABLE DES MATIÈRES

MATHÉMATIQUES Pages

N. Efimoff. Recherches sur certains invariants arithmétiques d'un point parabolique d'une surface 855

I. Privaloff. Membre correspondant de l'Académie. Sur les intégrales du type de Cauchy 859

P. Novikoff. Sur les projections de certains ensembles mesurables *B* . . 864

B. A. Fuchs. Sur les représentations pseudo-conformes d'un domaine sur lui-même avec un point fixe sur la frontière 866

B. Gnedenko. Sur les lois limites de la théorie des probabilités 870

PHYSIQUE

A. Achieser. Über einige Eigenschaften des Elektronengases in einem Magnetfeld 874

M. A. Markov. Rôle of the Zero-State of Radiation Oscillators in the Approximations of Quantum Electrodynamics 879

B. L. Granovsky. Diffusion of Ions in a Discharge and the Initial Rate of Gas Deionization. I 883

B. L. Granovsky. Diffusion of Ions in a Discharge and the Initial Rate of Gas Deionization. II 888

Ch. M. Fatalïev. Effect of Weak Magnetic Field upon Plasma 891

N. Perfilov. Désintégration de l'uranium par les neutrons 896

V. L. Ginsburg. Quantum electrodynamics. II 899

PHYSIQUE TECHNIQUE

A. G. Arenberg. Self-Induction and Capacity of a Hertz Vibrator and its Equivalency to a Closed Oscillatory Circuit 904

B. A. Wwedensky. Corresponding Member of the Academy, and **E. N. Maizels.** Electromagnetic Field of Paraboloid of Revolution Reflectors 908

CHIMIE PHYSIQUE

A. A. Grünberg und P. M. Filinow. Über die Anwendung künstlicher Radioelemente zur Aufklärung einiger Fragen der Komplexchemie . 912

CHIMIE

P. Schorigin, Mitglied der Akademie, und **N. N. Makarowa-Semljanskaja.** Der Übergang von Kohlenhydraten zu karbozyklischen Verbindungen. I. Die Umwandlung der Glukose in Phenol 915

Pages

B. L. Moldavskij and **T. V. Nisovkina.** Study of the Isomerization of Butanes and their Equilibrum Ratios 919
I. G. Druzhinin. Conversion of Astrakhanite 921

HYDROCHIMIE

A. V. Trofimov. Oxidizing Activity and pH of Brown Sediments of the Barentz Sea 925
A. V. Trofimov. Relation between the Chlorinity and the Density of Caspian Sea Water 929

CRISTALLOGRAPHIE

D. P. Grigorjev and **I. I. Shafranovsky.** A Crystallographic Investigation of Artificial Mullite 933

HISTOLOGIE

Sh. D. Galustian. General Character of the Growth and Transformations of the Elements of the X-Rayed Thymus outside of the Organism . . 935
Sh. D. Galustian. In vitro Cultivation of the Epithelium of the Thyroid Gland 937

GÉNÉTIQUE

P. D. Trebushenko. Inheritance of Anthocyanine Coloration in Apple-Tree (A Contribution to the Problem of Breeding Red-Fruited and Red-Fleshed Varieties of Apples) 939
N. E. Zaikovskaja. Reduction Division in Interspecific Hybrids in the Genus *Beta* L. II. Cytological Evidence on the Question on Experimental Synthesis of the Species *Beta trigyna* W. et K. ($2n = 54$) 944

PHYSIOLOGIE VÉGÉTALE

A. S. Morosov. Effect of Temperature upon the Reversible Activity of Invertase in Forage Grasses as Dependent on their Cold and Heat Resistance 949
J. V. Rakitin and **L. M. Jarkovaja**. Relationship between the Reaction of Medium and the Activity Growth Substances 952

PHYSIOLOGIE DES ANIMAUX

Ch. S. Koshtojanz. Corresponding Member of the Academy, and **R. L. Mitropolitanskaja.** Further Investigations on the Mechanism of the Formation and Destruction of the Chemical Factors of Nervous Stimulation 955
A. M. Rjabinovskaja. Influence of Sodium Fluoride, Sodium Maleinate, Glutamic Acid and Glyceraldehyde on Muscular Activity 958

MÉCANIQUE DU DÉVELOPPEMENT

I. Schaxel und **G. Schneider.** Die Einwirkung des Nervensystems auf die Regeneration der Gliedmassen beim Axolotl 962
A. I. Machabeli. Heterogeneous Induction of the Extremity in *Triton Vittatus* 967
N. A. Manuilova. Exhaustion and Restoration of the Inductive Power of the Eye Cup in Repeated Inductions in *Anura* 972
T. A. Sikharulidze. Studies on Lens-Forming Properties of Body Epithelium in *Triton Vittatus* 975
S. V. Emelianov. Sequence in the Ontogenetic Appearance of Vertebral Arches in Teleosts and the Omission of Chondral Stages in their Development 978

MORPHOLOGIE ÉVOLUTIONNAIRE

I. F. Trukhanov. Phenomenon of Wing Reduction in Some *Hemiptera* . 982

PALÉONTOLOGIE

N. V. Pimenova. Age of «Poltava» Sands in the Vicinity of Shesterintsy (Kiev Province) 985

Rédacteur en chef A. Fersman. Rédacteur S. Bernstein.

En composition 9/VI 1939. En impression 25/VII 1939. Nombre de feuilles d'impression 8 1/2. Nombre de caractères par feuille d'impression 65.000. Pages 853—988. Format du papier 72×100 cm. Tirage 1000. Glavlit A 15918. Numéro d'édition 2078. Numéro typographique 1280.

Imprimerie № 16, Moscou, Trekhproudnij péréoulok, 9.

Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1939. Volume XXIII, № 9

MATHÉMATIQUES

RECHERCHES SUR CERTAINS INVARIANTS ARITHMÉTIQUES D'UN POINT PARABOLIQUE D'UNE SURFACE

Par N. EFIMOFF

(Présenté par N. A. Kolmogoroff, de l'Académie, le 23.III.1939)

Soit M_0 un point régulier d'une surface analytique U. Considérons la transformation de Gauss de la surface U en E, où E est la sphère unitaire; soit M un point arbitraire de U et M' le point qui lui correspond sur E; fixons ensuite la direction des rotations positives autour de la normale à U au point M_0. Si M, tout en se déplaçant sur un petit contour simple entourant M_0 sur la surface U, le décrit une fois dans la direction positive, M' peut faire k tours autour du point M'_0 sur la sphère E dans la même direction ou dans la direction opposée. Le nombre k peut être supérieur à 1 seulement dans le cas d'un point parabolique M_0, c'est-à-dire dans le cas où la courbure de Gauss au point M_0 est nulle. Ce nombre a été signalé par S. Cohn-Vossen (1) comme un invariant arithmétique caractérisant la structure de la surface au voisinage de M_0. Convenons d'appeler i n d i c e d'un point M_0 le nombre $+k$, si M' fait k tours autour de M'_0 dans la direction positive, et le nombre $-k$, si M' décrit k tour dans la direction négative. L'indice peut prendre les $+1, 0, -1, -2, \ldots$ et il sera défini au point M_0, s'il ne passe pas par M_0 de ligne plane parabolique le long de laquelle les plans tangents à U coïncident.

Le théorème suivant est évident.

T h é o r è m e I. *L'indice d'un point reste invariant au cours de chaque déformation continue du voisinage d'un point parabolique isolé.*

C'est S. Cohn-Vossen qui a indiqué à l'auteur le théorème I et c'est lui qui a proposé d'étudier si cet indice est invariant par rapport à une déformation discrète, et de résoudre la question en quelle mesure l'hypothèse que le point parabolique est isolé est essentielle dans ce théorème. Il faut remarquer que le théorème I cesse d'être évident si l'on suppose qu'il passe par le point considéré une ou plusieurs lignes paraboliques, car en un certain moment de la déformation l'une quelconque de ces lignes peut devenir plane et telle que les plans tangents à la surface coïncideront le long de cette ligne; alors l'indice deviendra indéterminé.

Dans la suite nous donnons les résultats d'une étude concernant l'indice et d'autres invariants arithmétiques*.

---

* Tout récemment l'auteur a fait connaissance d'un article de H. Schilt (2) consacré aux points paraboliques isolés. On trouve dans cet article un résultat, exprimé dans notre texte par le théorème VIII, et certains corollaires de ce théorème.

Dans la suite nous désignerons par $U(M_0)$ un voisinage du point M_0 sur U, par $\tau(M_0)$ — le plan tangent à $U(M_0)$ au point M_0, par $\operatorname{ind} M_0$ — l'indice au point M_0, par p — l'ordre de contact de $U(M_0)$ et de $\tau(M_0)$, par p_i et p_s — les bornes inférieure et supérieure des valeurs de p pour toutes les surfaces $U'(M'_0)$ isométriques à $U(M_0)$, et enfin par m — le nombre des branches (réelles) de la courbe d'intersection de $U(M_0)$ et $\tau(M_0)$ ayant des directions différentes au point singulier M_0.

Il est facile à démontrer les théorèmes suivants:

Théorème II. *Si* $\operatorname{ind} M_0 = 0$, *il passe par* M_0 *une ligne parabolique.*

Théorème III. *Si* $m = p+1$, *le point parabolique est isolé.*

Théorème IV. p_s *existe, si la métrique de* $U(M_0)$ *n'est pas euclidienne.*

Dans la suite nous excluons de nos considérations les surfaces à métrique euclidienne.

Théorème V. *Si* $m > 1$, *on a* $p = p_s$.

Théorème VI. *On a pour chaque surface* $U(M_0)$, $p_i = 1$.

Théorème VII. $|\operatorname{ind} M_0| \leqslant p$.

On obtient comme corollaire des théorèmes VI et VII le

Théorème VIII. *Quelle que soit la surface* $U(M_0)$, *il existe une surface* $U'(M'_0)$ *qui lui est isométrique et telle que* $\operatorname{ind} M'_0$ *est égal à l'un des trois nombres*: -1, 0, $+1$.

Le théorème VIII résoud par la négative la première question posée par S. Cohn-Vossen.

Convenons de dire que $U(M_0)$ est une surface en forme de s e l l e, si $m > 1$ et la courbe d'intersection de $U(M_0)$ et $\tau(M_0)$ ne contient pas de branches (réelles) à tangentes communes au point M_0.

Théorème IX. *Si les surfaces en forme de selle* $U(M_0)$ *et* $U'(M'_0)$ *peuvent être transformées l'une en l'autre par une déformation continue au cours de laquelle m reste supérieur à* 1, *on a* $\operatorname{ind} M_0 = \operatorname{ind} M'_0$ *.

On prend pour base de la démonstration de ce théorème le théorème V, en vertu duquel dans la déformation indiquée l'ordre de contact de la surface et du plan tangent $\tau(M_0)$ reste invariable, ainsi que la proposition suivante.

Supposons que $U(M_0)$ est déterminée en coordonnées cartésiennes par l'équation

$$z = f(x, y)$$

et que d'ailleurs M_0 coïncide avec l'origine des coordonnées et $\tau(M_0)$ avec le plan (x, y). En posant $p = n-1$, écrivons le développement de $f(x, y)$ en série de Taylor au voisinage du point M_0 sous la forme

$$f(x, y) = f^{(n)}(x, y) + f^{(n+1)}(x, y) + \ldots$$

où

$$f^{(n)}(x, y) = \frac{1}{n!}\left(a_1 x^n + \left[\frac{n}{1}\right] a_2 x^{n-1} y + \ldots + a_{n+1} y^n\right).$$

Supposons maintenant que la matrice

$$\left\| \begin{matrix} a_1 a_2 \ldots a_{n-1} \\ a_2 a_3 \ldots a_n \\ a_3 a_4 \ldots a_{n+1} \end{matrix} \right\| \quad (n \geqslant 4)$$

---

* Ici et dans la suite la première partie de l'équation de la surface que l'on déforme est supposée une fonction continue du paramètre de déformation, ainsi que ses dérivées partielles jusqu'à l'ordre $p+1$ par rapport aux coordonnées.

est de rang $=3$. Alors le paraboloïde osculateur de la surface $U(M_0)$ défini au moment initial par l'équation

$$z = f^{(n)}(x, y),$$

n'éprouve aucune déformation quelle que soit la déformation continue de la surface $U(M_0)$ conservant l'ordre de contact avec $\tau(M_0)$. (D'ailleurs, pour démontrer le théorème IX il faut tenir compte non seulement des propositions signalées, mais encore de certaines propriétés de la suite de Sturm).

Théorème X. *Si la ligne d'intersection de $U(M_0)$ et de $\tau(M_0)$ ne possède pas au point singulier M_0 de tangentes multiples, une déformation continue quelconque de la surface $U(M_0)$ conserve son ordre de contact avec le plan tangent $\tau(M_0)$.*

Il suit des théorèmes I et VIII, ainsi que des théorèmes VI et X qu'en général il existe pour chaque voisinage $U(M_0)$ du point parabolique M_0 des voisinages isométriques $U'(M_0')$, qui ne peuvent pas être obtenus de $U(M_0)$ par une déformation continue (même pour les voisinages infiniment petits).

Théorème XI. *Si $p > 1$ et si la ligne d'intersection de $U(M_0)$ et de $\tau(M_0)$ possède au point singulier M_0 une tangente double, la surface $U'(M_0')$ que l'on obtient de $U(M_0)$ par une déformation continue, au cours de laquelle l'ordre de contact de $U(M_0)$ et $\tau(M_0)$ varie d'une façon monotone, possède avec $\tau'(M_0')$ l'ordre de contact qui est au plus égal à 1; si $p > 2$ et s'il existe une tangente triple, l'ordre de contact de $U'(M_0')$ et de $\tau'(M_0')$ est au plus égal à 2; en général, si $p > n$ et s'il existe une tangente $(n+1)$, on a $p' \leqslant n$.*

En s'appuyant sur les théorèmes IX et X, on peut établir le

Théorème XII. *L'indice d'une surface $U(M_0)$ (non nécéssairement en forme de selle), pour laquelle la ligne d'intersection de $U(M_0)$ et $\tau(M_0)$ n'a pas de tangentes multiples au point M_0, reste invariable dans chaque déformation continue.*

En concluant, nous voulons signaler quelques résultats de caractère plus particulier.

Supposons que $U(M_0)$ est rapportée aux coordonnées de Gauss (u^1, u^2) et que $\overline{N}_0$ est le vecteur unitaire de la normale au point M_0. Prenons sur $\tau(M_0)$ près de M_0 un point arbitraire P et menons en ce point une perpendiculaire ν à $\tau(M_0)$. Soit M le point d'intersection de cette perpendiculaire avec la surface $U(M_0)$ et $K(M)$ — la courbure de Gauss au point M. Prenons sur cette perpendiculaire ν un segment de longueur $|K|$ à partir du point P et dans la direction de $\overline{N}_0$, si $K(M) > 0$; dans la direction opposée, si $K(M) < 0$. Soit $\overline{M}$ l'extremité de ce segment. Quand le point P se déplace sur le plan $\tau(M_0)$, le point $\overline{M}$ décrit une certaine surface passant par M_0. Soit $U_k(M_0)$ cette surface. Si g_{ij} est le tenseur métrique de la surface $U(M_0)$, G_{ij} et B_{ij} sont les deux vecteurs fondamentaux de la surface $U_k(M_0)$, alors on a au point M_0 les égalités:

$$G_{ij} = g_{ij} + K_i K_j,$$

$$B_{ij} = \frac{w}{W} K_{1ij}$$

où $w = \sqrt{\operatorname{Det} g_{ij}}$, $W = \sqrt{\operatorname{Det} G_{ij}}$; K_i et $K_{/ij}$ sont les dérivées covariantes de la courbure pas rapport à la metrique de la surface $U(M_0)$. Il en suit, en particulier, que la courbure de Gauss $\overline{K}(M_0)$ de la surface $U_k(M_0)$ est un invariant par rapport aux déformations de la surface $U(M_0)$.

Si M_0 se trouve à l'origine des coordonnées et si $\tau(M_0)$ coïncide avec le plan (x, y), la surface $U(M_0)$ aura pour équation $z=f(x, y)$.

Pour $p=2$ le développement de $f(x, y)$ en série de Taylor au voisinage de M_0 aura la forme

$$f(x, y)=f^{(3)}(x, y)+f^{(4)}(x, y)+\ldots$$

Designons par D le discriminant de la forme $f^{(3)}(x, y)$. Un calcul direct prouve que l'on a l'égalité

$$D=\overline{K}(M_0).$$

Il en résulte qu'au point singulier M_0 l'ordre de multiplicité des tangentes à la ligne d'intersection de $U(M_0)$ et $\tau(M_0)$ reste le même dans chaque déformation qui conserve $p=2$ (ce fait est bien connu pour le cas où $p=1$).

En partant du théorème III, on peut établir que si $U(M_0)$ et $U'(M'_0)$ sont deux surfaces en forme de selle isométriques ayant avec leurs plans tangents l'ordre de contact au plus égal à 3, alors ind $M_0=$ind M'_0.

Il faut remarquer que la notion de surface en forme de selle dans la définition que nous avons donnée est plus restreinte que dans la définition ordinaire.

Université d'État
à Voronezh.

Manuscrit reçu
le 14. IV. 1939.

LITTÉRATURE CITÉE

[1] D. Hilbert u. S. Cohn-Vossen, Anschauliche Geometrie, S. 180.
[2] H. Schilt, Compositio Mathem., 5 (1937).

Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1939. Volume XXIII, № 9

MATHÉMATIQUES

SUR LES INTÉGRALES DU TYPE DE CAUCHY

Par I. PRIVALOFF, membre correspondant de l'Académie

J'ai démontré dans mon article [1] la proposition suivante:

1. Si le contour rectifiable L possède au point x_0 une tangente déterminée, si la fonction sommable $f(x)$ est continue en ce point x_0 et l'intégrale singulière (valeur principale de Cauchy) $\frac{1}{2\pi i}\int_L \frac{f(x)\,dx}{x-x_0}$ existe, on a la formule:

$$\frac{1}{2\pi i}\int_L \frac{f(x)\,dx}{x-z} \to \frac{1}{2\pi i}\frac{f(x)\,dx}{x-x_0} \pm \frac{1}{2}f(x_0), \qquad (I)$$

où le point z tend vers x_0 le long d'un chemin quelconque non tangent à L (le signe $+$ correspond au cas où z reste à l'intérieur de L, le signe $-$ au cas où z est extérieur à L).

Signalons maintenant la proposition suivante facile à démontrer:

2. Si L est un contour rectifiable ayant une tangente déterminée en chaque point, sauf peut être en un nombre fini, et $F(z)$ est une fonction holomorphe et projectivement bornée à l'intérieur du contour L (ou hors de L) qui tend suivant les normales vers des valeurs limites $F(x)$ formant une fonction continue sur L et ayant des discontinuités réparables aux points exceptionnels, alors ces valeurs limites existeront pour tous les points de L et quand on suivra un chemin quelconque non-tangent ou même tangent à L.

En effet, désignons par D le domaine intérieur au contour L (ou bien extérieur) et supposons d'abord que $F(z)$ est bornée dans le domaine D.

En faisant une représentation conforme de D sur le cercle unitaire du plan ζ, désignons par $\Phi(\zeta)$ la fonction transformée; comme elle est holomorphe et bornée à l'intérieur du cercle $|\zeta|<1$, elle aura en chaque point ξ de la circonférence $|\xi|=1$, sauf aux points exceptionnels en nombre fini, une valeur limite $\Phi(\xi)$ le long d'un chemin orthogonal à la circonférence, et d'ailleurs $\Phi(\xi)$ est continue sur la circonférence $|\xi|=1$ et possède des discontinuités réparables aux points exceptionnels. Dans ce cas la fonction $\Phi(\zeta)$ est représentable par une intégrale de Cauchy

$$\Phi(\zeta)=\frac{1}{2\pi i}\int_{|\xi|=1}\frac{\Phi(\xi)\,d\xi}{\xi-\zeta}$$

ou, ce qui est le même, par une intégrale de Poisson

$$\Phi(re^{i\varphi}) = \frac{1}{2\pi}\int_0^{2\pi} \Phi(e^{i\theta})\,\frac{1-r^2}{1+r^2-2r\cos(\theta-\varphi)}\,d\theta; \qquad (1)$$

donc, $\Phi(\zeta)$ est continue dans le cercle fermé $|\zeta| \leqslant 1$ et par cette raison $F(z)$ est continue dans le domaine fermé $\overline{D}$, c'est-à-dire $F(z)$ tend vers une limite $F(x)$ quand $z \to x$ d'une façon quelconque.

Il est évident que le cas où la fonction $F(z)$ est projectivement bornée peut être réduit au cas considéré par une transformation linéaire.

Supposons maintenant que L est un contour partiellement uni (stückweise-glatt), c'est-à-dire formé d'un nombre fini d'arcs ayant une tangente continue. Nous allons nommer angle du contour au point M l'angle fait au point M par le vecteur tangent quand un point décrit la courbe L dans la direction positive; sa valeur sera désignée par $\Pi\theta$, où $-1 < \theta < +1$. Ainsi en un point du contour, où il existe une tangente déterminée, on a $\theta = 0$, et en un point angulaire θ diffère de zéro.

Le cas des points de rebroussement ($\theta = \pm 1$) est exclu de nos considérations.

Le théorème 1 et la proposititon auxiliaire 2 permettent de démontrer le théorème.

3. Si le contour L est partiellement uni et sans points de rebroussement et si la fonction $f(x)$ définie sur L vérifie la condition de Lipschitz

$$|f(x_1) - f(x_2)| < K\,|x_1 - x_2|^{\alpha}, \quad 0 < \alpha \leqslant 1\ (^1),$$

on a la formule:

$$\frac{1}{2\pi i}\int_L \frac{f(x)\,dx}{x-z} \to \frac{1}{2\pi i}\int_L \frac{f(x)\,dx}{x-x_0} \pm \frac{1 \mp \theta}{2} f(x_0) \qquad (II)$$

où le point z tenu vers un point arbitraire x_0 du contour le long d'un chemin quelconque (les signes supérieurs correspondent au cas où z est intérieur à L, les signes inférieurs — au cas où il est extérieur).

En effet, en remarquant que $\frac{1}{2\pi i}\int_L \frac{dx}{x-x_0} = \frac{1+\theta}{2}$, nous pouvons écrire le second membre de (II) dans la forme:

$$\frac{1}{2\pi i}\int_L \frac{f(x)\,dx}{x-x_0} \pm \frac{1 \mp \theta}{2} f(x_0) = \frac{1}{2\pi i}\int_L \frac{f(x)-f(x_0)}{x-x_0}\,dx + \frac{1+\theta}{2} f(x_0) \pm \frac{1 \mp \theta}{2} f(x_0). \qquad (2)$$

Ainsi le second membre de la formule (II) peut être présenté dans la forme:

$$\frac{1}{2\pi i}\int_L \frac{f(x)-f(x_0)}{x-x_0}\,dx + f(x_0) \qquad (3)$$

ou bien dans la forme

$$\frac{1}{2\pi i}\int_L \frac{f(x)-f(x_0)}{x-x_0}\,dx. \qquad (4)$$

Comme la fonction $f(x)$ vérifie la condition (1) de Lipschitz, il est facile à voir que les formules (3) et (4) présentent des fonctions continues du point x_0 partout sur L. D'autre part, en vertu du théorème 1, le second membre de la formule (II) présente les valeurs limites suivant les normales de l'intégrale du type de Cauchy (de l'intérieur où de

l'extérieur de L) en tous les points non-angulaires, et d'ailleurs, ainsi que nous venons de signaler, ces valeurs limites sont continues sur tout le contour, les points angulaires y compris.

En appliquant la proposition auxiliaire 2, nous verrons que le théorème 3 est vrai, si nous démontrons que l'intégrale du type de Cauchy est projectivement bornée (2). A cet effet nous posons $x - z = re^{i\chi}$ et nous aurons:

$$F(z) = \frac{1}{2\pi i} \int_L \frac{f(x)\,dx}{x - z} = \frac{1}{2\pi i} \int_L \frac{f_1(s)\, e^{i(\varphi - \chi)}}{r}\, ds$$

où $f_1(s) = f[x(s)]$.

Dans cette dernière expression nous séparons les parties réelle et imaginaire en substituant $\mu(s) + i\nu(s)$ au lieu de $f_1(s)$ et nous aurons:

$$F(z) = \frac{1}{2\pi i} \int_L \frac{\mu(s)\cos(\varphi - \chi)}{r}\, ds - \frac{1}{2\pi} \int_L \frac{\mu(s)\sin(\varphi - \chi)}{r}\, ds +$$

$$+ \frac{1}{2\pi} \int_L \frac{\nu(s)\cos(\varphi - \chi)}{r}\, ds - \frac{1}{2\pi i} \int_L \frac{\nu(s)\sin(\varphi - \chi)}{r}\, ds =$$

$$= \frac{1}{2\pi i} [P_1(x, y) + iQ_1(x, y) + iP_2(x, y) - Q_2(x, y)] \qquad (5)$$

ou bien

$$F(z) = \frac{1}{2\pi i} [P_1(x, y) + iQ_1(x, y) + i(P_2(x, y) + iQ_2(x, y))]$$

où l'on a posé

$$\left.\begin{aligned} P_1(x, y) &= \int_L \frac{\mu(s)\cos(\varphi - \chi)}{r}\, ds = \int_L \frac{\mu(s)\sin(n, r)}{r}\, ds, \\ Q_1(x, y) &= \int_L \frac{\mu(s)\sin(\varphi - \chi)}{r}\, ds = \int_L \frac{\mu(s)\cos(n, r)}{r}\, ds \end{aligned}\right\} \qquad (5')$$

et de même pour P_2 et Q_2 en remplaçant $\mu(s)$ par $\nu(s)$ où (n, r) est l'angle de la normale intérieure au point s et du vecteur $\bar{r}$ allant du point s vers le point z et pris avec un signe convenable. Comme $Q_1(x, y)$ comme potentielle de double couche est une fonction harmonique à l'intérieur du contour L (ou hors de L), la fonction $P_1(x, y) + iQ_1(x, y)$ sera holomorphe et projectivement bornée à l'intérieur de L (ou hors de L). Donc, en supposant d'abord $\nu(s) \equiv 0$, nous démontrerons, en vertu des propositions 1 et 2, le théorème 3 pour $F_1(z) = P_1 + iQ_1$; en supposant ensuite $\mu(s) \equiv 0$, nous démontrerons le théorème 3 pour $F_2(z) = P_2 + iQ_2$, donc il sera démontré pour

$$F(z) = \frac{1}{2\pi i} [F_1(z) + iF_2(z)].$$

On peut compléter le théorème 3 par la proposition suivante:

4. Dans les conditions du théorème 3, les valeurs limites de la formule (II) vérifient la condition de Lipschitz pour le même exposant α, si $\alpha < 1$ et avec un exposant aussi proche de 1 que l'on voudra si $\alpha = 1$.

Cette dernière proposition a été établie dans mon article précédent (3) pour le cas d'une circonférence.

La méthode qui suit permet d'étendre le procédé que j'ai appliqué à une circonférence au cas d'un contour L quelconque partiellement uni et sans point de rebroussement.

Démonstration. Il suit des formules (3) et (4) que le problème consiste en l'étude d'une intégrale de la forme

$$g(x_\sigma) = \int_L \frac{f(x) - f(x_\sigma)}{x - x_\sigma} dx \qquad (6)$$

qu'il faut considérer comme fonction de x_σ.

En donnant à σ un accroissement $\Delta\sigma$ nous aurons

$$g(x_\sigma + \Delta x_\sigma) = \int_L \frac{f(x) - f(x_\sigma + \Delta x_\sigma)}{x - x_\sigma - \Delta x_\sigma} dx,$$

donc

$$g(x_\sigma + \Delta x_\sigma) - g(x_\sigma) = \int_L \left[\frac{f(x) - f(x_\sigma + \Delta x_\sigma)}{x - x_\sigma - \Delta x_\sigma} - \frac{f(x) - f(x_\sigma)}{x - x_\sigma} \right] dx. \qquad (7)$$

Integrons d'abord de $\sigma - \varepsilon$ à $\sigma + \varepsilon$, où $\varepsilon = 3\,|\Delta\sigma|$.

En tenant compte de la condition (1) nous trouverons que cette partie de l'intégrale (7) est en valeur absolue inférieure à

$$K_1 \int_{\sigma-\varepsilon}^{\sigma+\varepsilon} [|s - \sigma - \Delta\sigma|^{\alpha-1} + |s - \sigma|^{\alpha-1}]\, ds < K_2 |\Delta\sigma|^\alpha \qquad (8)$$

parce que

$$1 \geqslant \frac{|x_1 - x_2|}{|s_1 - s_2|} > c > 0. \qquad (9)$$

Il reste à effectuer l'intégration le long de l'arc L_ε obtenu de L quand on enlève l'arc $(\sigma - \varepsilon,\ \sigma + \varepsilon)$. Transformons d'abord la fonction sous le signe d'intégrale (7) en lui donnant la forme

$$|f(x) - f(x_\sigma + \Delta x_\sigma)] \frac{\Delta x_\sigma}{(x - x_\sigma)(x - x_\sigma - \Delta x_\sigma)} - [f(x_\sigma + \Delta x_\sigma) - f(x_\sigma)] \frac{1}{x - x_\sigma}. \qquad (10)$$

En remarquant que

$$\left| \int_L \frac{dx}{x - x_\sigma} \right| < C,$$

nous faisons la conclusion: l'intégrale de la fonction (10) prise le long de la ligne L_ε sera en valeur absolue inférieure à

$$K_3 |\Delta\sigma| + K_4 |\Delta\sigma|^\alpha, \quad \text{si } \alpha < 1 \qquad (11)$$

et si $\alpha = 1$ elle sera inférieure à

$$K_5 |\Delta\sigma| \ln \frac{1}{|\Delta\sigma|} + K_6 |\Delta\sigma| < K_7 |\Delta\sigma|^{1-\eta} \qquad (12)$$

où $\eta > 0$ est aussi petit que l'on veut. Pour obtenir les évaluations (11) et (12) nous avons tenu compte des inégalités (1) et (9). En réunissant les évaluations (8) et (11) et respectivement (8) et (12) nous voyons que

$$|g(x_\sigma + \Delta x_\sigma) - g(x_\sigma)| < C_1 |\Delta\sigma|^\alpha, \quad \text{si } \alpha < 1$$

et

$$|g(x_\sigma + \Delta x_\sigma) - g(x_\sigma)| < C_2 |\Delta\sigma| \ln \frac{1}{|\Delta s|} < C_3 |\Delta\sigma|^{1-\eta},$$

où $\eta > 0$ est aussi petit que l'on veut si $\alpha = 1$.

Les dernières inégalités peuvent être remplacées par des inégalités qui leur équivalent:

$$|g(x_\sigma + \Delta x_\sigma) - g(x_\sigma)| < C_1' |\Delta x_\sigma|^\alpha, \quad \text{si } \alpha < 1$$

et

$$|g(x_\sigma + \Delta x_\sigma) - g(x_\sigma)| < C_3' |\Delta x_\sigma|^{1-\eta} \quad \text{si } \alpha = 1.$$

Donc, le théorème 4 est complétement démontré.

En concluant signalons une proposition concernant la potentielle de double couche que l'on obtient des théorèmes 3 et 4 si l'on sépare dans la formule (II) les parties réelles des parties imaginaires et si l'on pose $\nu(s) \equiv 0$.

5. Si le contour L est partiellement uni et sans points de rebroussement et la fonction $\mu(s)$ définie sur L vérifie la condition de Lipschitz

$$|\mu(s_1) - \mu(s_2)| < K|s_1 - s_2|^{\alpha} \quad (0 < \alpha \leqslant 1)$$

on a les formules

$$\left.\begin{aligned} Q_1(x,y) &= \int_L \mu(s) \frac{\cos(n,r)}{r} ds \to \int_L \mu(s) \left(\frac{\cos(n,r)}{r}\right)_\sigma ds \pm \pi(1 \mp \theta), \\ P_1(x,y) &= \int_L \mu(s) \frac{\sin(n,r)}{r} ds \to \int_L \mu(s) \left(\frac{\sin(n,r)}{r}\right)_\sigma ds \end{aligned}\right\} \quad \text{(III)}$$

où le point (x, y) tend vers le point σ le long d'un chemin quelconque [les signes supérieurs correspondent au cas où (x, y) est à l'intérieur de L, les signes intérieurs — où il est hors de L]; d'ailleurs, les valeurs limites des formules (III) vérifient encore la condition de Lipschitz avec le même exposant α si $\alpha < 1$ et avec un exposant aussi proche de 1 que l'on voudra si $\alpha = 1$.

Autrement dit, dans le cas d'un contour partiellement uni L et d'une densité vérifiant la condition de Lipschitz d'ordre α, la potentielle de double couche $Q_1(x, y)$ et sa fonction harmonique conjuguée $P_1(x, y)$ ont pour tous les chemins (intérieurs ou extérieurs) et en tous les points du contour L des valeurs limites exprimées par les formules (III) et d'ailleurs ces valeurs limites vérifient aussi la condition de Lipschitz d'ordre α si $\alpha < 1$ et d'ordre aussi proche de 1 que l'on voudra si $\alpha = 1$.

Manuscrit reçu
le 15. IV. 1939.

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Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1939. Volume XXIII, № 9

MATHÉMATIQUES

SUR LES PROJECTIONS DE CERTAINS ENSEMBLES MESURABLES *B*

Par P. NOVIKOFF

(Présenté par I. M. Vinogradow, de l'Académie, le 22. IV. 1939)

Kunugui (1) a démontré le théorème suivant:

Si E est un ensemble plan du type G_δ, l'ensemble des points x_0 tels que les droites $x=x_0$ coupent E en un ensemble fermé est toujours un ensemble CA.

Il suit de ce théorème que la projection d'un ensemble plan du type G_δ, qui est coupé par chaque droite parallèle a l'axe OY en un ensemble fermé, est toujours mesurable B.

Nous allons démontrer le théorème suivant qui est une généralisation du théorème cité:

Théorème. *Si E est un ensemble plan mesurable B, l'ensemble des points x_0 tels que la droite $x=x_0$ coupe E en un ensemble fermé est toujours mesurable B.*

Nous allons nous appuyer sur le lemme suivant.

Lemme. Si E est un ensemble plan mesurable B, l'ensemble des points appartenant à E et situés sur les droites $x=x_0$ coupant E en un ensemble fermé est toujours un ensemble CA.

Démonstration. Soit E la fermeture de l'ensemble E le long des droites paralléles a l'axe OY. Il est connu que $\overline{E}$ est un ensemble A (2). Soit W la projection de l'ensemble $\overline{E}-E$ sur l'axe OX et S l'ensemble des points du plan OXY dont les projections appartiennent à CW. Il est évident que S est un ensemble CA.

Donc, $P=SE$ est aussi un ensemble CA. Il est facile à voir que P est l'ensemble dont il s'agit dans l'énoncé du lemme.

Démonstration du théorème. Sans restreindre la géneralité, on peut supposer que l'ensemble plan mesurable B est situé dans le carré unitaire du plan OXY. Désignons par I_{nk} la bande fermée du carré pour laquelle $\frac{k-1}{n} \leqslant y \leqslant \frac{k}{n}$. Soit Q_{nk} la projection de $I_{nk}E$ sur l'axe OX et E_{nk} l'ensemble de tous les points de l'ensemble I_{nk}, dont les abscisses appartiennent à Q_{nk}. Posons

$$E_n = \sum_{k=1}^{n} E_{nk}.$$

Il est évident que E_{nk} et E sont des ensembles A.

Désignons par E l'ensemble $\prod_{n=1}^{\infty} E_n$. Considérons l'ensemble $E_n - E$. En vertu du second théorème généralisé sur la séparabilité multiple (3), il existe un système dénombrable d'ensembles $CA: H_1, H_2, \ldots, H_n, \ldots$ tels que $H_n \supset E_n - E$ et $\prod_{n=1}^{\infty} H_n = 0$.

Soit U_{nk} l'ensemble de tous les points de I_{nk} qui appartiennent aux droites entièrement situées dans l'ensemble $H_n + P$ dans la bande I_{nk}. Il est évident que tous les U_{nk} sont des ensembles CA, ainsi que les ensembles $U_n = \sum_{k=1}^{n} U_{nk}$. Comme

$$H_n + P \supset U_n, \quad U_n \supset P \text{ et } \prod_{n=1}^{\infty} H_n = 0$$

on a $\prod_{n=1}^{\infty} U_n = P$.

Considérons les ensembles $\overline{U}_n U_1 U_2 \ldots U_n$; chacun des ensembles $\overline{U}_n$ est coupé par les parallèles à l'axe OY en un nombre fini de segments et $\overline{U}_1 \supset \overline{U}_2 \supset \ldots U_n \supset \ldots$ donc la projection de la partie commune de tous les $\overline{U}_n$ sur l'axe OX coïncide avec la partie commune des projections de ces ensembles sur l'axe OX. Mais ces derniers sont des ensembles CA, donc la projection de $\prod_{n=1}^{\infty} \overline{U}_n = P$ sur OX est un ensemble CA.

Corollaire 1. *La projection sur l'axe OX d'un ensemble plan mesurable B coupé par chaque droite parallèle à OY en un ensemble fermé est un ensemble mesurable B.*

Corollaire 2. *La famille des ensembles plans mesurables B coupés par chaque parallèle à OY en un ensemble fermé coïncide avec la famille des fermetures le long des parallèles à OY des systèmes dénombrables de courbes mesurables B.*

Manuscrit reçu
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Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1939. Volume XXIII, № 9

MATHÉMATIQUES

SUR LES REPRÉSENTATIONS PSEUDO-CONFORMES D'UN DOMAINE SUR LUI-MÊME AVEC UN POINT FIXE SUR LA FRONTIÈRE

Par B. A. FUCHS

(Présenté par M. N. Kotchine, de l'Académie, le 29. IV. 1939)

1. Soit D un domaine univalent de l'espace des deux variables complexes z^1, z^2. On peut alors, ainsi qu'il a été fait par M. S. Bergmann, définir dans ce domaine une fonction qu'il a nommé «Kernfunktion»:

$$K(z, \bar{z}) = k(z^1, z^2, \bar{z}^1, \bar{z}^2) = \sum_{k=0}^{\infty} |\varphi_k(z^1, z^2)|^2 \tag{1}$$

où les fonctions φ_k forment un système orthogonal normal et complet (pour les fonctions f régulières dans D et telles que $\int_D |f|^2 d\omega < \infty$, où $d\omega$ est l'élément de volume). La fonction peut aussi être définie au moyen d'un certain problème minimal et on établit ainsi qu'elle ne dépend que du domaine D. On définit au moyen de $K(z, \bar{z})$ une métrique riemannienne pour le domaine D invariante pour les représentations pseudo-conformes du domaine D (1):

$$ds^2 = T_{m\bar{k}} dz^m d\bar{z}^k; \quad T_{m\bar{k}} = \frac{\partial^2 \lg K}{\partial z^m \partial \bar{z}^k} \tag{2}$$

[(2) est une forme d'Hermite positive; la représentation pseudo-conforme $z^{*k} = f^k(z^1, z^2)$ est une représentation biunivoque du domaine D effectuée au moyen de deux fonctions analytiques].

Soient (2,3)

$$u^k(z, \bar{t}) = T^{k\bar{r}}_{(t)} \frac{\partial \lg M(z, \bar{t})}{\partial \bar{t}^r} = \frac{L^k(z, \bar{t})}{K(z, \bar{t})} \tag{3}$$

ce que l'on appelle les coordonnées représentatives du domaine D par rapport au point $t(t^1, t^2)$. Ici $M(z, \bar{t}) = \frac{K(z, \bar{t})}{K(t, \bar{t})}$ est ce que l'on appelle la fonction minimale de D (1). L'indice (t) ici et dans la suite montre que la valeur correspondante est calculée au point t. Soit

$$\delta z^k = \xi^k(z^1, z^2)\, \delta p \tag{4}$$

une transformation infiniment petite des mouvements analytiques de la géométrie invariante (2) (toutes les représentations pseudo-conformes du domaine D sur lui-même présentent des mouvements de cette nature).

J'ai démontré dans un article précédent (4) que les fonctions ξ^k peuvent toujours être présentées dans la forme [le point $P(t^1, t^2)$ qui figure dans l'article cité est ici remplacé par le point t]:

$$\xi^r(z) = \xi^s(t)\,\tau^{r\bar{e}}\,T^{(t)}_{s\bar{e}} - \overline{\xi^s(t)}\,\tau^{r\bar{e}}\,\frac{\partial^2 \lg M(z,\bar{t})}{\partial \bar{t}^s\,\partial t^e} - \frac{\overline{\partial \xi^s(t)}}{\partial t^e}\,\tau^{r\bar{e}}\,\frac{\partial \lg M(z,\bar{t})}{\partial \bar{t}^s}. \qquad (5)$$

Cela étant, on peut en tenant compte de (3), (4) et de l'égalité (4) de l'article cité (écrite pour le point t) obtenir la relation

$$\delta[u^s(z,\bar{t})] = \xi^s(t)\,\delta p + \xi^s_{,r}(t)\,u^r(z,\bar{t})\,\delta p - \overline{\xi^r(t)}\,\frac{\partial u^s(z,\bar{t})}{\partial \bar{t}^r}\,\delta p \qquad (6)$$

où $\xi^s_{.r}$; $u^s_{.r}$ sont les dérivées covariantes des valeurs correspondantes au point t. Remarquons que dans la métrique (2) on a $u^s_{,\bar{r}} = \dfrac{\partial u^s(z,\bar{t})}{\partial \bar{t}^r}$.

Soit ensuite E une certaine suite de points t_i (t_i^1, t_i^2) du domaine D qui converge vers un certain point frontière $G(t^1, t^2)$ de ce domaine. Convenons d'appeler les valeurs α, β_k l'ordre relatif des valeurs $K(z,\bar{t})$, $L^k(z,\bar{t})$ pour la suite E s'il existe dans D et peuvent être obtenues uniformément dans chaque domaine D_1 (le domaine D_1 ainsi que sa frontière est entièrement contenu dans D) des limites qui ne sont pas identiquement nulles:

$$\lim_{i\to\infty}[K(t_i\,\bar{t}_i)]^{-\alpha}\,K(z,\bar{t});\quad \lim_{i\to\infty}[T^{k\bar{k}}_{(t_i)}\,K(t_i,\bar{t}_i)]^{\beta_k}\,L^k(z,\bar{t}). \qquad (7)$$

Remarquons à propos de ces définitions que si les valeurs $K(t_i,\bar{t}_i)$, $T^{k\bar{k}}(t_i\,\bar{t}_i)$. $K(t_i,\bar{t}_i)$ tendent vers l'infini, on a toujours α, $\beta_k \leqslant \frac{1}{2}$. Cela résulte des inégalités

$$\left.\begin{aligned} |K(z,\bar{t})|^2 &\leqslant K(z,\bar{z})\,K(t,\bar{t}),\\ |L^k(z,\bar{t})|^2 &\leqslant K(z,\bar{z})\,K(t,\bar{t})\,T^{k\bar{k}}(t,\bar{t}) \end{aligned}\right\} \qquad (8)$$

que l'on obtient en appliquant l'inégalité de Schwarz aux expressions (1) et (3).

Pour le cas des domaines où l'allure de la fonction $K(z,\bar{z})$ sur la frontière a été étudié par M. Bergmann(1) (et si d'ailleurs on fait certaines hypothèses sur la suite E) la valeur $T^{k\bar{k}}R$ dans l'expression (7) peut être remplacée par une certaine puissance de K. Dans ces cas on a $\lim_{i\to\infty} K(t_i,\bar{t}_i) = \infty$.

Remarquons enfin que si D est une hypersphère ou bien un bicylindre, on a $\alpha = \beta_k = 0$.

2. Supposons que pour les composantes $\xi^k(z)$ de la transformation infiniment petite d'un groupe de mouvements analytiques du domaine D il existe une suite E de points $t_i(t_i^1, t_i^2)$ qui converge vers un point frontière G et que

$$\lim_{i\to\infty}\xi^k(t_i) = 0. \qquad (9)$$

Ceci est évidemment vrai si les $\xi^k(z)$ sont régulières et s'annullent au point G (par exemple, cela a lieu pour une transformation infiniment petite d'un certain groupe continu de représentations pseudo-conformes

du domaine D sur lui-même, si l'on suppose que ces transformations sont régulières au point frontière G et possèdent ce dernier pour point fixe). Dans ce cas on peut démontrer le théorème suivant:

Théorème. *Soient*

$$\delta z^k = \xi^k (z^1, z^2)\, \delta p$$

des transformations infiniment petites d'un groupe de mouvements analytiques de l'espace telles que l'on a

$$\lim_{i\to\infty} \xi^k (t_i^1, t_i^2) = 0$$

(*soit* $E(t_i^1, t_i^2)$ *une certaine suite de points* t_i *du domaine* D *qui converge vers un point frontière* G).

Alors si les conditions suivantes sont vérifiées:

A) les ordres α, β_k *des valeurs* $K(z, \bar{t})$ *et* $L^k(z, \bar{t})$ *existent pour* E;

B) il existe

$$\lim_{i\to\infty} \xi^s_{,\,r} \frac{(KT^{s\bar{s}})^{-\beta_s}}{(KT^{r\bar{r}})^{-\beta_r}} = \alpha^s_r, \quad (\alpha^s_r \neq 0);$$

C) les limites suivantes existent et sont nulles *

$$\lim_{i\to\infty} \frac{[T^{s\bar{s}} K]^{-\beta_s}}{K^{-\alpha}} \xi^s (t_i), \quad \lim_{i\to\infty} \frac{[T^{k\bar{k}} K]^{-\beta_k}}{K^{\alpha-1}} S^{\frac{1}{2}}_{sk} \xi^s (t_i);$$

dans ces conditions il existe des fonctions

$$v^s = \lim_{i\to\infty} \frac{[T^{s\bar{s}} K]^{-\beta_s}}{K^{-\alpha}} u^s (z, \bar{t_i}) = F^s (z^1, z^2).$$

Elles définissent une certaine représentation analytique dont le déterminant n'est pas identiquement nul et qui transforme D *en un certain domaine* R. *Dans ce domaine aux transformations du groupe des mouvements analytiques obtenues par les transformations infinimemt petites* $\delta z^k = \xi^k \delta p$ *correspondent des représentations linéaires et homogènes du domaine* K *en lui-même.*

On a

$$S_{sk} = T^{k\bar{e}} T^{m\bar{k}} \Big[(KK_{m\bar{e}s\bar{s}} - K_{\bar{e}} K_{s\bar{s}m} - K_m K_{s\bar{s}\bar{e}} + K_{s\bar{s}} R_{m\bar{e}}) - \frac{1}{K^2} (KK_{sm} - K_s K_m)(KK_{\bar{s}\bar{e}} - K_{\bar{s}} K_{\bar{e}}) - K^2 T^{p\bar{q}} \frac{\partial T_{pe}}{\partial t^s} \frac{\partial T_{mq}}{\partial t^s} - K \left(K_s \frac{\partial T_{m\bar{e}}}{\partial \bar{t}^s} + K_{\bar{s}} \frac{\partial T_{m\bar{e}}}{\partial t^s} \right) \Big].$$

La démonstration de ce théorème est basée sur la relation (6).

Dans son énoncé nous ne faisons aucune hypothèse sur le caractère de la frontière du domaine et sur la disposition de la suite E par rapport à cette frontière.

Si l'on considère les cas étudiés par Bergmann dans ses travaux consacrés à l'analyse de la fonction $K(z, \bar{z})$ sur la frontière du domaine [1], on arrive aux résultats suivants (nous conservons la terminologie et les notations de cet article cité):

1) Si pour la suite E et pour le domaine D on a les conditions «d'approximation A^I» vers un point du 3-ième ordre (le point G doit

* On ne fait pas ici la sommation suivant les indices s et k.

être de cette nature), alors les conditions B) et C) de notre théorème peuvent être remplacées par la condition suivante: la limite

$$\lim_{i\to\infty} \frac{\partial \xi^s(t_i)}{\partial t_i^e} \frac{(KT^{s\bar{s}})^{-\beta_s}}{(KT^{e\bar{e}})^{-\beta_e}}$$

existe et $\xi^s(t_i)$ est un infiniment petit d'ordre qui n'est pas inférieur à celui de $\rho^{\overline{\frac{3}{2}}}$ (où ρ est la projection de Gt_i sur la normale intérieure de l'hypersurface frontière au point G).

2) Si pour la suite E et pour le domaine D on a les conditions «d'approximation A^V» vers un point du 2-ième ordre (le point G doit être de cette nature), alors les conditions B) et C) de notre théorème peuvent être remplacées par la condition suivante: la limite

$$\lim_{i\to\infty} \frac{\partial \xi^s(t_i)}{\partial t_i^e} \frac{(KT^{s\bar{s}})^{-\beta_s}}{(KT^{e\bar{e}})^{-\beta_e}}$$

existe et l'infiniment petit $\xi^s(t_i)$ est d'ordre qui n'est pas inférieur à celui de ρ^2 (ρ ayant le sens précédemment expliqué).

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à Voronezh.

Manuscrit reçu
le 29. IV. 1939.

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Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1939. Volume XXIII, № 9

MATHÉMATIQUES

SUR LES LOIS LIMITES DE LA THÉORIE DES PROBABILITÉS

Par B. GNEDENKO

(Présenté par A. N. Kolmogoroff de l'Académie, le 26.IV. 1939)

Le but de la présente Note est d'indiquer des conditions nécessaires et suffisantes pour la convergence des lois de distribution des sommes de variables indépendantes vers la loi limite; je me propose de donner des conditions encore plus simples que celles qui ont été indiquées dans une Note précédente (1). La simplification est obtenue grâce à un certain changement de la position du problème.

Dans cette Note, comme dans la Note précédente, nous allons considérer une suite de séries

$$x_{n1},\quad x_{n2},\quad \ldots,\quad x_{nk_n} \tag{1}$$

de variables aléatoires indépendantes dans chaque série. Ces variables aléatoires sont nommées constantes à la limite si l'on peut trouver des constantes b_{nk} telles que les différences $x_{nk}-b_{nk}$ peuvent être négligées quand on fait un passage à la limite, c'est-à-dire telles que pour chaque $\varepsilon>0$ on a uniformément par rapport à k $(1\leqslant k\leqslant k_n)$ la relation)

$$P\{|x_{nk}-b_{nk}|>\varepsilon\}\underset{n\to\infty}{\longrightarrow}0.$$

Il est évident que si les b_{nk} existent, on peut prendre pour b_{nk} les médianes des valeurs x_{nk}, c'est-a-dire les constantes m_{nk} telles que

$$P\{x_{nk}<m_{nk}\}\leqslant\frac{1}{2}\leqslant P\{x_{nk}\leqslant m_{nk}\}.$$

Désignons la loi de distribution de la variable aléatoire x_{nk} par $F_{nk}(x)$; alors pour que les valeurs x_{nk} soient constantes à la limite, il est nécessaire et suffisant que l'on ait la relation

$$\sup_{1\leqslant k\leqslant k_n}\int_{-\infty}^{\infty}\frac{x^2}{1+x^2}\,dF_{nk}(x+m_{nk})\underset{n\to\infty}{\longrightarrow}0.$$

Nous allons chercher des conditions nécessaire et suffisantes pour que l'on puisse trouver des constantes A_n pour lesquelles les lois de distribution des sommes

$$\sigma_n=x_{n1}+x_{n2}+\ldots+x_{nk_n}-A_n \tag{2}$$

convergent vers la loi limite (en chaque point de continuité de cette dernière). Je donne ces conditions nécessaires et suffisantes dans deux formes différentes.

Théorème I. *Pour que l'on puisse trouver des constantes A_n telles que les lois de distribution des sommes* (2) *de variables aléatoires indé-*

pendantes constantes à la limite convergent vers la loi limite, il est nécessaire et suffisant qu'il existe une fonction $G(u)$ $(G(-\infty)=0)$ *telle que*

$$1.\quad \sum_{k=1}^{k_n} \int_{-\infty}^{u} \frac{x^2}{1+x^2}\, dF_{nk}(x+\alpha_{nk}) \underset{n\to\infty}{\to} G(u)$$

aux points de continuité de $G(u)$.

$$2.\quad \sum_{k=1}^{k_n} \int_{-\infty}^{\infty} \frac{x^2}{1+x^2}\, dF_{nk}(x+\alpha_{nk}) \underset{n\to\infty}{\to} G(+\infty)$$

ou

$$\alpha_{nk} = m_{nk} + \int_{|x|<1} x\, dF_{nk}(x+m_{nk}).$$

Remarque. Si

$$\lim_{\varepsilon\to 0} \lim_{n\to\infty} \sum_{k=1}^{k_n} \Big(\int_{|x|<\varepsilon} x\, dF_{nk}(x+m_{nk}) \Big)^2 = 0 \qquad (3)$$

les quantités α_{nk} du théorème I peuvent être choisies égales aux médianes $(\alpha_{nk}=m_{nk})$. Si la loi limite n'a pas de composante gaussienne [c'est-à-dire si $G(+0)-G(-0)=0$], ou bien si les lois $F_{nk}(x)$ sont symétriques par rapport aux médianes, la relation (3) est vérifiée.

Théorème II. *Pour que l'on puisse choisir les constantes* A_n *de telle manière que les lois de distribution des sommes* (2) *de variables aléatoires indépendantes constantes à la limite convergent vers la loi limite, il est nécessaire et suffisant qu'il existe des fonctions* $M(u)$ [*pour* $u<0$, $M(-\infty)=0$] *et* $N(u)$ [*pour* $u>0$, $N(+\infty)=0$] *et une constante* a *telles que*

1. *Aux points de continuité des fonctions* $M(u)$ *et* $N(u)$

$$\sum_{k=1}^{k_n} F_{nk}(u+m_{nk}) \to M(u) \quad \textit{pour} \quad u<0.$$

$$\sum_{k=1}^{k_n} (F_{nk}(u+m_{nk})-1) \to N(u) \quad \textit{pour} \quad u>0.$$

$$2.\quad \lim_{\varepsilon\to 0} \lim_{n\to\infty} \sum_{k=1}^{k_n} \Big\{ \int_{|x|<\varepsilon} x^2 dF_{nk}(x+m_{nk}) - \Big(\int_{|x|<\varepsilon} x\, dF_{nk}(x+m_{nk}) \Big)^2 \Big\} = a^2.$$

Les constantes A_n peuvent être choisies égales à

$$A_n = \sum_{k=1}^{k_n} \Big\{ m_{nk} + \int_{-\infty}^{\infty} \frac{x}{1+x^2}\, dF_{nk}(x+m_{nk}) \Big\}.$$

Le logarithme de la fonction caractéristique de la loi limite est donné par la formule de P. Lévy.

$$\lg \varphi(t) = -\frac{a^2}{2} t^2 + \int_{-\infty}^{-0} \Big(e^{iut} - 1 - \frac{iut}{1+u^2} \Big)\, dM(u) + \int_{+0}^{\infty} \Big(e^{iut} - 1 - \frac{iut}{1+u^2} \Big)\, dN(u).$$

Comme conséquence des théorèmes précédents nous donnons la forme la plus générale de la loi des grands nombres pour les sommes de variables indépendantes.

Théorème III. *Pour qu'il existe des constantes A_n telles que pour chaque $\varepsilon > 0$*

$$P\{|x_{n1}+x_{n2}+\ldots+x_{nk_n}-A_n| \geqslant \varepsilon\} \underset{n\to\infty}{\to} 0$$

il est nécessaire et suffisant que l'on ait

$$\sum_{k=1}^{k_n} \int_{-\infty}^{\infty} \frac{x^2}{1+x^2}\, dF_{nk}(x+m_{nk}) \underset{n\to\infty}{\to} 0.$$

Remarquons que si l'on suppose que les valeurs x_{nk} peuvent être négligées à la limite, on peut partout dans les énoncés des théorèmes I — III remplacer les valeurs α_{nk} par

$$\int_{|x|<\tau} x\, dF_{nk}(x),$$

où τ est une constante arbitraire. Nous obtenons donc en passant les théorèmes que j'ai exposés dans mes Notes précédentes (1) et (2).

En ce qui concerne les démonstrations des théorèmes I — III faisons la remarque suivante. Si pour un certain choix des constantes A_n les lois de distribution des sommes (2) convergent vers la loi limite, les deux conditions suivantes sont toujours vérifiées.

1. Il existe une constante C qui ne dépend pas de n et telle que

$$\sum_{k=1}^{k_n} \int_{-\infty}^{\infty} \frac{x^2}{1+x^2}\, dF_{nk}(x+m_{nk}) < C.$$

2. Il existe pour chaque $\varepsilon > 0$ un nombre $b(\varepsilon)$ qui ne dépend pas de n et tel que pour certains $x = x(n)$ on a

$$F_n(x+l(\varepsilon)) - F_n(x) \geqslant 1-\varepsilon.$$

On désigne ici par $F_n(x)$ la loi de distribution de la somme

$$S_n = x_{n1}+x_{n2}+\ldots+x_{nk_n}. \tag{4}$$

Cela permet d'appliquer le lemme suivant.

Lemme I. Si les conditions 1) et 2) sont vérifiées, les lois de distribution $F_n(x)$ des sommes (4) et les lois indéfiniment divisibles $\Phi_n(x)$, dont les logarithmes $\Psi_n(t)$ des fonctions caractéristiques sont donnés par la formule

$$\Psi_n(t) = \sum_{k=1}^{k_n} \left\{ im_{nk}t + \int_{-\infty}^{\infty} (e^{itx}-1)\, dF_{nk}\left(x+m_{nk}+\right.\right.$$

$$\left.\left. + \int_{|x|<\varepsilon} x\, dF_{nk}(x+m_{nk})\right)\right\}$$

où ε est une constante positive aussi petite que l'on veut, jouissent de cette propriété que leur différence $F_n(x)-\Phi_n(x)$ converge en mesure vers zéro quand $n\to\infty$.

Ce lemme est une généralisation d'un théorème connu de G. M. Bowli (3); il montre que quand le nombre des termes de la somme (4) croît, ces sommes peuvent être approchées par des sommes de variables aléatoires, indéfiniment divisibles, l'ordre d'approximation étant aussi grand que l'on veut. On obtient du lemme I le

L e m m e II. Pour que pour un certain choix des constantes A_n les lois de distribution des sommes (2) de variables aléatoires indépendantes constantes à la limite convergent vers la loi limite, il est nécessaire et suffisant que les lois de distribution indéfiniment divisibles dont les logarithmes des fonctions caractéristiques sont définis par la formule

$$\lg \varphi_n(t) = -iA_n t + \sum_{k=1}^{k_n} \left\{ im_{nk}t + \right.$$

$$\left. + \int_{-\infty}^{\infty} (e^{itx} - 1)\, dF_{nk}\left(x + m_{nk} + \int_{|x|<1} x\, dF_{nk}(x + m_{nk}) \right) \right\},$$

convergent vers la loi limite.

Les lois limites pour les deux suites coïncident.

Le lemme II joint au théorème II de ma Note précédente couduit au résultat cherché.

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Manuscrit reçu
le 3. V. 1939.

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Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1939. Volume XXIII, № 9

PHYSIK

ÜBER EINIGE EIGENSCHAFTEN DES ELEKTRONENGASES IN EINEM MAGNETFELD

Von A. ACHIESER

1. Es ist bekannt, dass infolge der Quantelung der Elektronenenergie in einem Magnetfeld ein aus freien Elektronen bestehendes Gas ein diamagnetisches Moment bekommt, das für schwache Felder $\mu H \ll kT$ $\left(\mu = \frac{e\hbar}{mc}\right)$ einem Drittel des infolge des Spins entstehenden paramagnetischen Momentes gleich ist (1). Die Quantelung der Energie führt auch zu einer Änderung des Widerstandes des Metalls im Magnetfeld.

Es ist daher nicht ohne Interesse die Frage nach der Rolle der Quantelung bei niedrigen Temperaturen (wenn die Bedingung $\mu H \ll kT$ nicht erfüllt ist) zu untersuchen.

Um die Rechnungsmethode zu klären, wollen wir zunächst die Berechnung des magnetischen Momentes betrachten. Die Aufgabe besteht, bekanntlich, in der Berechnung der Summe

$$\Omega = -kT \sum \ln\left[1 + e^{\frac{\varepsilon_0 - \varepsilon}{kT}}\right]$$

die auf alle Eigenwerte der Energie des Elektrons erstreckt ist; das magnetische Moment M ist mit Ω durch die Beziehung $M = -\frac{\partial \Omega}{\partial H}$ verknüpft. Die Energieeigenwerte des Elektrons im magnetischen Felde sind, falls man den Spin berücksichtigt,

$$\varepsilon = n\mu H + \frac{p^2}{2m},$$

wo p die Komponente des Elektronenimpulses längs des Feldes bedeutet $(n = 0, 1, 2, \ldots)$.

Beachtet man die Gewichte der verschiedenen Zustände und führt dimensionslose Grössen

$$\eta = \frac{\mu H}{\varepsilon_0}, \quad \tau = \frac{kT}{\varepsilon_0}, \quad u = \frac{p}{\sqrt{m\varepsilon_0}}$$

(ε_0 ist chemisches Potential) ein, so ergibt sich für das auf die Volumeneinheit bezogene Ω der folgende Ausdruck:

$$\Omega = -\left(\frac{\sqrt{m\varepsilon_0}}{\hbar}\right)^3 \frac{\varepsilon_0 \tau \eta}{2\pi^2}\left(\frac{1}{2}\int_{-\infty}^{+\infty} \ln\left[1 + e^{\frac{1}{\tau}\left(1 - \frac{u^2}{2}\right)}\right] du + \right.$$

$$\left. + \sum_{n=1}^{\infty} \int_{-\infty}^{+\infty} \ln\left[1 + e^{\frac{1}{\tau}\left(1 - \eta n - \frac{u^2}{2}\right)}\right] du\right).$$

Nun wollen wir denjenigen Fall betrachten, wenn τ und $\eta \ll 1$ sind. Um die Summe zu berechnen, haben einige Autoren [1] die Euler-Maclaurinsche Summationsformel benutzt. Die Bedingung für die Anwendbarkeit dieser Formel ist $\eta \ll \tau$. In dem uns interessierenden Falle $\eta \gg \tau$ werden wir von der Poissonschen Summationsformel [2]

$$\frac{1}{2} f(0) + \sum_{n=1}^{\infty} f(n) = \int_0^{\infty} f(x)\,dx + 2 \sum_{n=1}^{\infty} \int_0^{\infty} f(x) \cos 2\pi n x\,dx$$

Gebrauch machen. Setzt man in dieser Formel

$$f(n) = \ln\left[1 + e^{\frac{1}{\tau}\left(1 - \eta n - \frac{u^2}{2}\right)}\right],$$

so ergibt sich

$$\Omega = -\left(\frac{\sqrt{m\varepsilon_0}}{\hbar}\right)^3 \frac{\varepsilon_0 \tau \eta}{2\pi^2} \left(\int_{-\infty}^{+\infty} du \int_0^{\infty} \ln\left[1 + e^{\frac{1}{\tau}\left(1 - \eta x - \frac{u^2}{2}\right)}\right] dx + \right.$$

$$\left. + 2 \sum_{n=1}^{\infty} \int_{-\infty}^{+\infty} du \int_0^{\infty} \ln\left[1 + e^{\frac{1}{\tau}\left(1 - \eta x - \frac{u^2}{2}\right)}\right] \cos 2\pi x\,dx \right).$$

Führt man die Bezeichnung

$$I_n(u) = \int_0^{\infty} \ln\left[1 + e^{\frac{1}{\tau}\left(1 - \eta x - \frac{u^2}{2}\right)}\right] \cos 2\pi n x\,dx$$

ein, so ergibt sich nach der partiellen Integration

$$I_n(u) = \begin{cases} \dfrac{1}{2\eta\tau} \displaystyle\int_{\frac{-1-\frac{u^2}{2}}{\tau}}^{\infty} \frac{\left(1 - \frac{u^2}{2} + \tau z\right)^2 e^z}{(e^z+1)^2}\,dz, & n = 0 \\ \dfrac{\eta}{4\pi^2 n^2 \tau} \dfrac{1}{e^{\frac{1}{\tau}\left(\frac{u^2}{2}-1\right)} + 1} - & \\ - \dfrac{\eta}{4\pi^2 n^2 \tau} Re\, e^{2\pi i n \frac{1-\frac{u^2}{2}}{\eta}} \displaystyle\int_{\frac{1-\frac{u^2}{2}}{\tau}}^{\infty} \frac{e^z}{(e^z+1)^2} e^{2\pi i n \frac{\tau z}{\eta}} dz, & n \neq 0. \end{cases}$$

Für $u > \sqrt{2}$ kann man annähernd $I_n = 0$ annehmen, während für $u < \sqrt{2}$ können wir die untere Integrationsgrenze $-\frac{1-\frac{u^2}{2}}{\tau}$ durch $-\infty$ ($\tau \ll 1$) ersetzen. Wir haben daher für $u < \sqrt{2}$

$$I_0(u) = \frac{1}{2\tau\eta}\left(1 - \frac{u^2}{2}\right)^2 + \frac{\pi^2}{6}\frac{\tau}{\eta},$$

$$I_n(u) = \frac{\eta}{4\pi^2 n^2 \tau} \frac{1}{e^{\frac{1}{\tau}\left(\frac{u^2}{2}-1\right)} + 1} - \frac{\cos\frac{2\pi n}{\eta}\left(1 - \frac{u^2}{2}\right)}{2n \operatorname{sh}\frac{2\pi^2 n\tau}{\eta}}.$$

Die Integration nach u gibt für $\eta \ll 1$

$$\int_{-\infty}^{+\infty} I_n(u)\,du = \frac{2\sqrt{2}\,\eta}{4\pi^2 n^2 \tau} - \frac{\sqrt{\eta}}{2\pi^{\frac{3}{2}}} \frac{\cos\left(\frac{2\pi n}{\eta} - \frac{\pi}{4}\right)}{\operatorname{sh}\frac{2\pi^2 n\tau}{\eta}}$$

und wir bekommen die Formel*

$$\Omega = -\left(\frac{\sqrt{m\varepsilon_0}}{\hbar}\right)^3 \frac{\varepsilon_0}{2\pi^2}\left(\frac{\sqrt{28}}{15}+\frac{\sqrt{2}\pi^2}{3}\tau^2+\frac{\sqrt{2}}{6}\eta^2-\right.$$
$$\left.-\tau\eta^{\frac{3}{2}}\sum_{n=1}^{\infty}\frac{\cos\left(\frac{2\pi n}{\eta}-\frac{\pi}{4}\right)}{n^{\frac{3}{2}}\operatorname{sh}\frac{2\pi^2 n\tau}{\eta}}\right).$$

Diese Formel ist richtig für beliebige τ und η, falls nur τ und $\eta \ll 1$ sind. Ist $\eta \ll \tau$, so kann man die unendliche Summe vernachlässigen; in diesem Fall ist

$$M = -\frac{\partial\Omega}{\partial H} = \frac{\sqrt{2}}{6\pi^2}\frac{\mu^2}{\varepsilon_0}\left(\frac{\sqrt{m\varepsilon_0}}{\hbar}\right)^3 H,$$

und dies ist ein bekanntes Resultat: $\frac{2}{3}$ des Paulischen Paramagnetismus. Ist dagegen $\eta \gg \tau$, so kann man sh durch sein Argument ersetzen. Behält man in Ω nur die η und $\eta^{\frac{1}{2}}$ proportionalen Glieder, so ergibt sich

$$M = \left(\frac{\sqrt{m\varepsilon_0}}{\hbar}\right)^3 \frac{\mu}{2\pi^2}\left(\frac{\sqrt{2}\,\eta}{3}-\frac{\eta^{\frac{1}{2}}}{\pi}\sum_{n=1}^{\infty}\frac{\sin\left(\frac{2\pi n}{\eta}-\frac{\pi}{4}\right)}{n^{\frac{3}{2}}}\right).$$

Für niedrige Temperaturen ist also das magnetische Moment eine komplizierte sich schnell ändernde Funktion des Feldes. Die Amplitude des «periodischen» Bestandteils des Momentes ist proportional $\eta^{\frac{1}{2}}$, d. h. grösser als das lineare Glied. Für nicht zu kleine η ist der periodische Teil bedeutender als der lineare. Der Maximalwert der unendlichen Summe ist gleich

$$\frac{1}{\sqrt{2}}\zeta\left(\frac{3}{2}\right) \sim 1.8.$$

2. Die Idee der Quantelung der Elektronenbewegung ist von Titeica (4) benutzt worden, um die Änderung des Widerstandes des Metalls im magnetischen Felde zu untersuchen. Titeica hat nämlich die Änderung des Widerstandes für schwache ($\mu H \ll kT$) und sehr starke ($\mu H > \varepsilon_0$) Felder gefunden; dabei hat er angenommen, dass die Energie des Elektrons dem Quadrat des Impulses proportional ist, und dass die Energie der Wechselwirkung zwischen dem Elektron und dem Gitter gleich D div $\vec{u}$ ($\vec{u}$ = Wärmeverschiebungsvektor) ist.

Man kann zeigen, dass für niedrige Temperaturen und Felder, für welche $\mu H \ll \varepsilon_0$ ist, der Widerstand in einer sehr komplizierten Weise von dem Felde abhängt, wobei auch hier eine charakteristische «Periodizität» zum Vorschein kommt.

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* Ein ähnlicher Ausdruck für die Summe Ω, die die magnetischen Eigenschaften von *Bi* bestimmt, ist nach einer anderen Methode in (3) berechnet.

Der allgemeine Ausdruck für den Widerstand in einem transversalen Feld hat die Form (4):

$$\rho_t = \frac{m^2 D^2}{32\pi^3 w e^2 \hbar^2 \nu^2 \delta} \frac{\mu H}{kT} \sum_{n,n'} \int_0^\infty \int_0^\infty df' \, df_z \frac{f'^3 f}{f_z} \cdot$$

$$\cdot \frac{W_{nn'} e^\xi}{(e^\xi - 1)^2} \left(\frac{1}{e^{\frac{x-\xi}{2}} + 1} - \frac{1}{e^{\frac{x+\xi}{2}} + 1} + \frac{1}{e^{\frac{x'-\xi}{2}} + 1} - \right.$$

$$\left. - \frac{1}{e^{\frac{x'+\xi}{2}} + 1} \right), \qquad n, n' = 0, 1, 2, \ldots$$

wobei

$$X = \frac{1}{kT} \left\{ \frac{\mu H}{2} (n + n' + 1) + \frac{\hbar^2 f_z^2}{8m} + \frac{m}{2 f_z^2} [\omega - \omega_0 (n' - n)]^2 - \varepsilon_0 \right\},$$

$$X' = \frac{1}{kT} \left\{ \frac{\mu H}{2} (n + n' + 1) + \frac{\hbar^2 f_z^2}{8m} + \frac{m}{2 f_z^2} [\omega + \omega_0 (n' - n)]^2 + \varepsilon_0 \right\}, \quad \xi = \frac{\hbar \omega}{kT}$$

ist und $\omega = wf$ die Phononenfrequenz, w — die Schallgeschwindigkeit, f' — die zu dem Magnetfeld senkrechte Komponente von $\vec{f}$, ν — die Elektronendichte, δ — das spezifische Gewicht und $\omega_0 = \frac{eH}{mc}$ bedeuten.

$W_{nn'}$ ist das Modulquadrat des Matrixelementes der Störungsenergie

$$W_{nn'} = \left| \int_{-\infty}^{+\infty} e^{i\tau_x z} \Phi_n(z) \Phi_{n'}(z + \tau_y) \, dz \right|^2,$$

$$\vec{\tau} = \sqrt{\frac{\hbar c}{eH}} \vec{f} \; *.$$

Wir wollen den Fall

$$H \gg \frac{c}{e\hbar} \left(\frac{kT}{w} \right)^2, \quad \mu H \ll \varepsilon_0$$

betrachten; dabei wird $|\vec{\tau}|$ der Grössenordnung nach $\ll 1$ sein $\left(f \right.$ hat für niedrige Temperaturen die Ordnung der Grösse $\left. \frac{kT}{\hbar w} \right)$. Wir können daher $W_{nn'}$ in eine Reihe nach τ_x und τ_y entwickeln und nur freie Glieder behalten. Es folgt $W_{nn'} = \delta_{nn'}$. Führt man ferner anstatt f_z und f' neue Veränderliche f und $t = \frac{f^2}{f_z^2}$ ein, so wird

$$\delta t = \frac{m^2 D^2}{32\pi^2 w e^2 \hbar^2 \nu^2 \delta} \frac{\mu H}{kT} \left(\frac{kT}{\hbar w} \right)^5 \int_0^\infty \frac{\xi^4 e^\xi \, d\xi}{(e^\xi - 1)^2} \int_1^\infty \frac{t - 1}{t^2} \cdot$$

$$\cdot \, dt \sum_{n=0}^{\infty} \left(\frac{1}{e^{\frac{1}{kT}\left[\mu H\left(n+\frac{1}{2}\right) + \lambda t - \varepsilon_0 - \frac{\hbar\omega}{2}\right]} + 1} - \frac{1}{e^{\frac{1}{kT}\left[\mu H\left(n+\frac{1}{2}\right) + \lambda t - \varepsilon_0 + \frac{\hbar\omega}{2}\right]} + 1} \right)$$

wobei $\lambda = m w \frac{2}{2}$ bedeutet.

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* $\Phi_n(z)$ ist die normierte Oszillatorenfunktion.

Benutzt man die Poissonsche Formel und beachtet, dass der Widerstand in der Abwesenheit des Magnetfeldes gleich $\rho_0 = \frac{5}{\pi^3} \frac{D^2 m^2}{w \delta e^2 \hbar^2 \nu^2} \left(\frac{kT}{\hbar w}\right)^5$ ist, so ergibt sich, dass

$$\rho_t = \frac{3}{4} \rho_0 \left\{ \ln \frac{\varepsilon_0}{e\lambda} + \sum_{n=1}^{\infty} (-1)^n \frac{4\pi^2 n \frac{kT}{\mu H}}{\operatorname{sh} \frac{2\pi^2 n kT}{\mu H}} A \cdot \left(\frac{\pi n kT}{\mu H}\right) L_t \left(\frac{2\pi n \varepsilon_0}{\mu H}, \frac{2\pi n \lambda}{\mu H}\right) \right\}$$

ist, wo

$$A(\alpha) = \frac{1}{5! \alpha} \int_0^{\infty} \frac{\xi^4 \sin \alpha \xi \, e^{\xi}}{(e^{\xi} - 1)^2} d\xi,$$

$$L_t(\alpha, \beta) = -\cos(\alpha - \beta) - (\cos \alpha - \beta \sin \alpha) \, Ci \, \beta + (\sin \alpha + \beta \cos \alpha) \left(\frac{\pi}{2} - Si \, \beta\right)$$

bedeuten. Für $\frac{2\pi n \lambda}{\mu H} \gg 1$ und $T \to 0$ folgt

$$\frac{\rho_t}{\rho_0} \sim \frac{3}{4} \left\{ \ln \frac{\varepsilon_0}{e\lambda} - \frac{1}{2\pi^2} \left(\frac{\mu H}{\lambda}\right)^2 \sum_{n=1}^{\infty} (-1)^n \frac{\cos \frac{2\pi n}{\mu H}}{n^2} (\varepsilon_0 - \lambda) \right\}.$$

Wenn $\lambda \ll \mu N$ ist und $T \to 0$, so ist

$$\frac{\rho_t}{\rho_0} \sim \frac{3}{4} \left(\ln \frac{\varepsilon_0}{e\lambda} - 2 \ln \frac{\mu H}{2\pi\lambda} \cos \frac{2\pi \varepsilon_0}{\mu H} \right).$$

Im Falle eines longitudinalen Feldes hat der Widerstand die Grösse

$$\rho_e = \frac{3}{2} \rho_0 \left\{ 1 + \sum_{n=1}^{\infty} (-1)^n \frac{4\pi^2 n \frac{kT}{\mu H}}{\operatorname{sh} \frac{2\pi^2 n kT}{\mu H}} A \left(\frac{\pi n kT}{\mu H}\right) L_e \left(\frac{2\pi n \varepsilon_0}{\mu H}, \frac{2\pi n \lambda}{\mu H}\right) \right\}$$

wo

$$L_e(\alpha, \beta) = \cos(\alpha - \beta) - \beta \sin \alpha \, Ci \, \beta - \left(\frac{\pi}{2} - Si \, \beta\right) \beta \cos \alpha$$

ist.

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Eingegangen d. 28. IV. 1939.

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Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1939. Volume XXIII, № 9

PHYSICS

RÔLE OF THE ZERO-STATE OF RADIATION OSCILLATORS IN THE HIGHER APPROXIMATIONS OF QUANTUM ELECTRODYNAMICS

By M. A. MARKOV

(Communicated by V. A. Fock, Member of the Academy, 27.IV.1939)

V. Pauli and M. Firz (1) investigated the radiation of long waves at the process of slowing down of the electric charge in a weak external field. The authors consider the dipole radiation and using the non-relativistic formulation of the problem introduce from the very start the finite dimension of the space occupied by the charge.

The cross-section obtained by the authors for the radiation of long waves appears essentially to contain a parameter characterizing the space dimensions of the charged body:

$$dq = \text{const}\,\frac{dE}{E}\left(\frac{E}{h\omega_1}\right)^C \qquad (1)$$

where ω_1 is the limit frequency, connected with the dimensions of the body (r_0) by the relation

$$\omega_1 = \frac{2\pi c}{r_0}, \qquad (2)$$

$$C = \frac{2}{3}\pi\frac{e^2}{hc}\left(\frac{v_0 - v_0'}{c^2}\right)^2, \qquad (3)$$

v_0 and v_0' are the initial and the final velocities of the charged body.

The authors came to a conclusion that the difficulty in question (for the point electron $-\omega_1 \to \infty$) is connected with the fundamental difficulties of quantum electrodynamics.

In reality the essential rôle of the parameter characterizing the dimensions of the body in the expression (1), seems to point toward the sources of these difficulties. Further inquiry shows, however, that the rôle of the space dimensions of the charge is not a direct one. The divergence arising from $(A)^2$ (A is a vector potential of the electron's own field) furnishes a representative instance to convince us that the divergence $\left(\int_0^\infty \omega\, d\omega\right)$ known to be connected with the zero-states of the radiation oscillators is converted in a finite member, if the same problem is considered for a distributed charge whereby the dimensions of the latter essentially enter into the final expression, viz. $\int_0^\infty \omega\, d\omega \to \frac{h}{mc^2} 4\pi \frac{e^2}{r_0^2}$. The simple fact here is that the radius of the

electron acts as a cutting factor in the frequency spectrum of the zero states of the oscillators. Very likely the same circumstance takes place also in the problem examined by Pauli and Firz. It is a matter for regret that the rôle of the zero-states of field oscillators in the higher approximation of the ordinary radiation theory, had not been studied as yet. In the generally accepted theory of radiation the higher approximations are known to diverge. Some of the divergent integrals appear to arise from the zero-states of these oscillators. And since Pauli and Firz apply a method which gives a solution containing higher approximations of the ordinary radiation theory, it is only natural to examine the rôle of the zero-states of the radiation oscillators in the present problem. In fact, as is shown below, all the particularities of the theory of radiation built up by Pauli and Firz are connected with the zero-states of the radiation oscillators and with the divergences arising from them.

To begin with consider a linear oscillating electron and write the function of Hamilton in the form:

$$H = \frac{1}{2}(p^2 + q^2)\, h\nu + \frac{1}{2}\sum_s (P_s^2 + Q_s^2)\, h\omega_s - \sum_s h^2\omega_s (\bar{\alpha}_s \bar{p})\, P_s;$$

the term $u = -\sum h^2\omega_s (\bar{\alpha}_s \bar{p})\, P_s$ will be considered a perturbation.

In the second approximation assume for the energy:

$$H''_{\substack{N=0 \\ n_s=0}} = -\frac{1}{3}\frac{h\nu e^2}{\pi m c^3}\int_0^\infty d\omega + \frac{h\nu^2}{\pi m c^3}\frac{1}{3}\int_0^\infty \frac{d\omega}{\omega + \nu} \tag{4}$$

(the oscillating electron being in its ground state $N_s = 0$, photons are absent, $n_s = 0$, the term A^2 is neglected).

The meaning of the integrals may be cleared at once by making the initial Hamilton function assume the canonical form in which the «bounded» photons are separated from the free ones ([1]).

$$\left.\begin{aligned} &\bar{q} = \bar{q}' - h\sum \bar{\alpha}_s Q_s; \quad \bar{p} = \bar{p}' \\ &Q_s = Q'_s; \qquad\qquad P_s = P'_s + (\bar{p}\bar{\alpha}_s). \end{aligned}\right\} \tag{5}$$

The first term must be interpreted as the magnetic energy of the point electron ([1]). The divergence of the second integral is bound up with the zero-states of radiation oscillators. In fact a term independent only of the impulse (p) of the electron is separated owing to the transformation (5) from the potential energy in the initial Hamilton function,

$$\frac{q^2}{2} \to \frac{q'^2}{2} + \frac{h^2}{2}\sum_s \bar{\alpha}_s^2 Q_s^2 + \dots \tag{6}$$

The second integral in the expression (4) (let it be denoted by I_2) takes its origin just in the member:

$$\text{const}\sum \alpha_s^2 Q_s^2. \tag{7}$$

I_2 is obtained as the average value of (7) over the zero-states of the field oscillator:

$$I_2 \to \text{const}\sum_{n_s=0} \alpha_s^2 \int \varphi_{n_s} Q_s^2 \varphi_{n_s} dQ_s = \text{const}\sum_{n_s=0} \alpha_s^2 (2n_s + 1) \tag{8}$$

Besides:

a) I_2 is independent of the state of the oscillating electron (of the number N);

b) at $n_s = 0$ (the absence of photons) I_2 does not become zero and the only reason for this is that the zero-motion of the electromagnetic field oscillators is available.

In the other formulae similar to (8) there may occur matrix elements, representing the average values of quantity $(Q_s)^k$ at the zero-state of the oscillators:

$$\int \varphi_{0_s}^* Q_s^k \varphi_{0_s} dQ_s. \tag{9}$$

It is obvious, that these average quantities do not vanish because of the zero-vibrations of these oscillators.

The matrix elements of the type (9) play a substantial part in the theory of Pauli and Firz. If indeed we are to compare the method of Pauli and Firz with the general theory of radiation, it will be evident that all the substantial distinctions of the Pauli's and Firz's theory are determined by the presence of the factor ([1]):

$$e^{-\sum_s w_s} \tag{10}$$

where

$$e^{-w_s} = (p_0'; 0_s \mid k \mid \bar{p}_0; 0_s) = \int \varphi_{0_s}^* e^{i(\bar{p}_0' - \bar{p}_0)\bar{\alpha}_s Q_s} \varphi_{0_s} dQ_s; \tag{11}$$

here $\bar{p}_0'$ and $\bar{p}_0$ are finite and initial values of the charge; w_s appears as a result of integration (11). Expression (11) represents the average value of the operator

$$e^{i(\bar{p}_0' - \bar{p}_0)\bar{\alpha}_s Q_s} \tag{12}$$

at the zero-state of the oscillator of the s kinds. It is only owing to the zero-oscillations of the electromagnetic field that expression (11) comes into being. The essential distinction of the Bloch-Nordsik-Pauli-Firz method from the ordinary theory of radiation consists in that the interaction between matter and radiation $\left(\text{i. e. the term } \frac{p}{m}\frac{e}{c}\bar{A}\right)$ is not accounted small (i. e. the member $-\sim \bar{p}\bar{A}$ is included in the undisturbed operators, the action of the outer field $v(r)$ is considered as a perturbation).

As an unperturbed function Pauli and Firz take the function:

$$\psi = e^{\frac{i\bar{p}\bar{r}}{\hbar}} e^{i\sum_s \bar{p}\bar{\alpha}_s Q_s} \prod_s \varphi_{n_s} \tag{13}$$

wich includes all the higher approximations of the ordinary theory of radiation arising from $\left(\frac{\bar{p}}{m}\frac{e}{c}\bar{A}\right)$. (Restrictions: unrelativistic case, dipole radiation). And convertely, in calculating ψ with the higher approximations of the ordinary radiation theory and considering $\frac{p}{m}\frac{e}{c}\bar{A}$ to be small, we obtain a complicated expression which can however be reduced to a more simple form:

$$\psi = \Phi_0 + \frac{i}{1!}\left(\sum \bar{p}\bar{\alpha}_s Q_s\right)\Phi_0 +$$

$$+ \frac{1}{2!}\left(i\sum_s \bar{p}\bar{\alpha}_s Q_s\right)^2 \Phi_0 + \ldots = \Phi_0 = \Phi_0' + \Phi_0'' + \ldots \tag{14}$$

Taking n terms of the series (4) as a unperturbed function and calculating the slowing down of the charge in a feeble external field $v(r)$ by the method of Pauli and Firz, we reach the result obtained by those authors in the n-th

approximation of the ordinary theory of radiation, with the only difference that the term corresponding at this approximation to the factor e^{-w_s} is obtained as a series of terms of the type (9):

$$e^{-w_s} \to 1 + i(\bar{p}_1 - \bar{p}_0)\bar{\alpha}_s \int \varphi^*_{0_s} Q_s \varphi_{0_s} dQ_s + \ldots \int \varphi_{0_s} Q_s^k \varphi_{0_s} dQ_s + \ldots \quad (15)$$

Thus the factor e^{-w_s} takes its origin, it has also been indicated above, from the zero-oscillations of the electromagnetic field.

In the light of this reasoning the results inferred from the theory of Pauli and Firz which appear unexpected at the first sight become clear. Thus, the rôle of the radius of the electron becomes trivial—its task is to cut off the spectrum of the zero-states which, as it is implied by the quantum electrodynamics of today, are inevitably present in the mechanism of the theory even if the radiation of very long waves is considered. In the given case the justification for the usual method of the radiation theory to be applied lies in the lawfullness of the expansion of (13) in series (14).

The method is obviously unlawful on the boundary of the «bremse spectrum», where the intensity of radiation $I^{\omega}_{\omega \to 0} = I_0$ is finite, while the number of photons tends to infinity, since for a large n_s we have (symbolically): $Q_s \approx \sqrt{\bar{n}_s} - \infty$. And as in general $Q_s \geqslant \sqrt{\frac{1}{2}}$ (due to zero-oscillations of the field), or, to be more precise:

$$Q_s \varphi_{n_s} = \sqrt{\frac{\bar{n}_s}{2}} \varphi_{n_s - 1} + \sqrt{\frac{\overline{n_s + 1}}{2}} \varphi_{n_s + 1} = \sqrt{\frac{1}{2}} \varphi_{n_s + 1},$$

then

$$\left(i \sum \bar{p} \bar{\alpha}_s Q_s\right)^k \geqslant \left(\frac{1}{2}\right)^{\frac{k}{2}} \left(i \sum \bar{p} \alpha_s\right)^k.$$

It is easy to see that these expressions diverge, i. e. $\left(\sum_s ip\alpha_s\right)^2 \approx$

$\approx \int_0^{\infty} \frac{d\omega}{\omega}$, and therefore, at the higher approximations, the ordinary method of the radiation theory can not be applied on account of the zero-state of the radiation oscillators. A more detailed report will be given elsewhere.

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Received
3. V. 1939.

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Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1939. Volume XXIII, № 9

PHYSICS

DIFFUSION OF IONS IN A DISCHARGE AND THE INITIAL RATE OF GAS DEIONIZATION. I

By B. L. GRANOVSKY

(Communicated by S. I. Wawilow, Member of the Academy, 6. V. 1939)

1. In calculating the fundamental quantities of the discharge plasma and in determining the deionization rate of gases under moderately low pressures, the diffusion theory ([1],[2],[3]) can be employed. To begin with it is necessary that the ion mean free path, λ_p, be considerably shorter than the linear dimensions of the discharge space, while volume recombination, most important at high pressures, must be quite insignificant. According to Schottky's ambipolar diffusion theory ([1]), the problem of ion and electron motion reduces to the customary diffusion equation:

$$\frac{\partial n}{\partial t} = D_a \Delta n + zn \tag{1}$$

where z is number of ion pairs generated per electron per second and D_a — ambipolar diffusion coefficient:

$$D_a = \frac{D_e b_p + D_p b_e}{b_e + b_p} \cong \frac{1}{3} \lambda_p \bar{c}_p \left(1 + \frac{U_e}{U_p}\right). \tag{2}$$

Here c_p is ion mean velocity, and U_e and U_p are electron and ion mean kinetic energies. In a steady discharge $\frac{\partial n}{\partial t} = 0$, while during deionization $z = 0$. For simple cases the solutions of equation (1) are well known.

If following Schottky we assume as boundary conditions that ion concentration at the walls of the tube is zero

$$n_w = 0 \tag{3}$$

then, for instance, in the case of an infinite cylindrical plasma of radius R the distribution of ion concentration, in a steady discharge as well as in a disintegrating plasma, is expressed by the equation

$$n(r) = n_0 J_0\left(\mu_1 \frac{r}{R}\right), \tag{4}$$

where n_0 is ion concentration at the axis, J_0 — Bessel function of zero order, $\mu_1 = 2.405\ldots$ the first root of the Bessel function J_0.

Under steady discharge conditions (3) imposes the following relation in the quantities z and D_a:

$$\sqrt{\frac{z}{D_a}} = \frac{\mu_1}{R}. \tag{5}$$

After arc extinction the same quantities will determine the rate of deionization:

$$-\frac{1}{n}\frac{\partial n}{\partial t}=\frac{1}{\tau}=D_a\left(\frac{\mu_1}{R}\right)^2. \qquad (6)$$

As a matter of fact, however, n_w is not 0; the ratio $\frac{n_w}{n_0}$ has various values depending upon pressure p and upon R. Therefore, the eigenwert $\mu \neq 2.405$, but it is likewise a function of p and R.

Thus, in any computations using the diffusion theory, particularly in calculating the deionization rate, besides the geometry of the tube it is necessary to know two more quantities: D_a and μ. They can be determined from the steady discharge.

2. An attempt can be made to determine μ theoretically, after more accurately investigating the boundary conditions.

This was done by W. Funk and R. Seeliger [4] who assumed that the density of the electron diffusion current to the walls $D_a\left(\frac{\partial n}{\partial r}\right)_w$ is equal to the density of the random electron current, less the current of reflected electrons:

$$D_a\left(\frac{\partial n}{\partial r}\right)_w=\frac{1}{4}n_w\bar{c}_e(1-\rho)$$

(ρ — coefficient of reflexion of electrons from the wall).

However, the fact that from an insulated wall nearly all of the electrons are reflected, had not been taken in account by these authors (in mercury vapour discharge, for instance, $1-\rho \cong \frac{1}{400}$); moreover in their numerical reckoning a mistake has been overlooked [in passing from formula (3) to formula (4) of their paper, the coefficient 4 had been omitted]. As a result, the correction term was underestimated by a factor of 1600 times, and they found that μ does not appreciably differ from 2.405.

In fact, the boundary conditions are better based on the motion of the ions, since the latter are totally absorbed by the walls. We shall use for this purpose the method shown in W. de Groot's paper [5] and applied by V. A. Fabrikant to the theory of diffusion of excited atoms [6].

Let us consider a stratum of positive space—charge at the walls (stratum A) and the adjoining stratum of plasma of the thickness $\frac{2}{3}\lambda_p$ (stratum B). 1 cm^2 of stratum B is entered by $-D_a\left(\frac{\partial n}{\partial r}\right)$ ions per second coming from the rest of the plasma.

From stratum B into stratum A there pass $\left(\frac{1}{2}n\bar{c}_p-\frac{1}{3}\lambda_p\bar{c}_p\frac{\partial n}{\partial r}\right)$ ions per second; in this formula the coefficients have twice the usual values, because we assume the ion velocity distribution as hemispherical toward the walls. Not a single ion is passing back into stratum B from stratum A, all of them absorbed by the walls. Finally, as a result of ionization by collision $\frac{2}{3}zn\lambda_p$ ions per second are forming in stratum B. The number of ions leaving a given stratum minus the number of ions entering it, is equal to the number of ions formed there:

$$\frac{2}{3}\lambda_p nz=-D_a\frac{\partial n}{\partial r}-\frac{1}{2}n\bar{c}_p+\frac{1}{3}\lambda_p\bar{c}_p\frac{\partial n}{\partial r}. \qquad (7)$$

It is here assumed that the thickness of both strata is small compared to the dimensions of the plasma, i. e. to R; therefore, in all the

terms of (7), n and $\frac{\partial n}{\partial r}$ are referred to the same $r = R$. Substituting D_a from (2), dividing by $\bar{c}_p$ and denoting $\frac{\bar{c}_p}{\lambda_p} = \nu_p$ (the average number of collisions per ion per second near to the wall), we find the unknown boundary conditions as

$$\frac{1}{3}\lambda_p \frac{U_e}{U_p}\left(\frac{\partial n}{\partial r}\right)_w = \left(\frac{2z}{3\nu_p} - \frac{1}{2}\right) n_w. \qquad (8)$$

If it be required that the solution of (4) satisfy (8) instead of (3), the equation for μ becomes:

$$\frac{{}_0J(\mu)}{\mu J_1(\mu)} = \frac{\lambda_p}{3R\left(\frac{1}{2} - \frac{2z}{3\nu_p}\right)} \frac{U_e}{U_p}. \qquad (9)$$

In place of (5) we then have

$$\sqrt{\frac{z}{D_a}} = \frac{\mu}{R}. \qquad (5^*)$$

When $\lambda_p \ll R$, $J_0(\mu)$ is comparatively small, it is possible to write approximately $\frac{J_0(\mu)}{J_1(\mu)} = 2.405 - \mu$. Then instead of (9) we obtain an algebraic equation from which we find

$$\mu = \frac{2.405}{1 + 0.82\frac{\lambda_p}{R}\left(\frac{1}{2} - \frac{2z}{3\nu_p}\right)\frac{U_e}{U_p}}. \qquad (10)$$

If we assume very approximately $\mu J_1(\mu) \approx \text{const} = 1.2$, then the ratio $\frac{n_w}{n_0} = J_0(\mu)$ may be presented simply as

$$\frac{n_w}{n_0} = J_0(\mu) \approx 0.4 \frac{\lambda_p}{R\left(\frac{1}{2} - \frac{2z}{3\nu_p}\right)} \frac{U_e}{U_p}. \qquad (11)$$

Thus, the value of $\frac{n_w}{n_0}$ is of the order of $\frac{\lambda_p}{R}$, as it was shown by V. de Groot **. Unfortunately, in the present state of the theory of ion motion in a plasma, it is impossible to make any general statement as to the values of ν_p and $\frac{U_e}{U_p}$.

It can only be observed that at low pressures the ratio $\frac{U_e}{U_p} \approx 2$ at the walls and increases with the pressure. A rough estimate indicates that with $p > 0.01$ mm Hg, the fraction $\frac{2z}{3\nu_p}$ is negligibly small, compared to $\frac{1}{2}$, while during deionization z is falling to zero.

3. To determine μ and D_a experimentally, the author in collaboration with E. V. Merzloukhova and E. A. Vintilina made systematic probe measurements in the positive column of a mercury vapour discharge under the conditions of the diffusion regime. The diameters of the discharge tubes were 65 and 105 mm, the pressure (referred to 0°C)—from 0.0023 to 0.065 mm Hg. The type of the discharge tube and the method of pressure control have been already described elsewhere [7]; the measurements were made with a cylindrical probe at the tube axis, and a plane probe with a guard-ring at the walls.

* I am obliged to V. A. Fabrikant for indicating the rôle of the last term and the consequent problems in the theory of probe measurements.

** Engel and Steenbeck [10] derived the same conclusion from calculations in which it was assumed that $\mu = 2.4$ (as it is known, this holds if $n_w = 0$), and a certain length for the free path was considered as average for both electrons and ions. In their calculations only the final result found already by V. de Groot, appears to be conclusive.

These experimental results will be published in detail in a coming paper in the «Journal of Exper. and Theoretical Physics»; we shall here merely reproduce the values of $\frac{n_w}{n_0}$, μ and D_a; $\frac{n_w}{n_0}$ is shown in Fig. 1 as a function of $\frac{\lambda_p}{R}$; λ_p has been taken from the expression $\lambda_p = \sqrt{2}\,\frac{\lambda_1}{p}$ where $\lambda_1 = 0.0015$ cm — free path of a mercury neutral molecule at a pressure of 1 mm Hg; p is the vapour pressure reduced to 0°C. We see that the rise of $\frac{n_w}{n_0}$ is slower than that of $\frac{\lambda_p}{R}$ $\left[\text{approximately as } \left(\frac{\lambda_p}{R}\right)^{0.7}\right]$; hence, and from formula (9), it follows that with decreasing pressure $\frac{U_e}{U_p}$ decreases too, as already mentioned above. The values of μ calculated from (4) are shown in Fig. 2; in spite of the conclusions of Funk and Seeliger (4), these values fall by 30%, when $\frac{\lambda_p}{R}$ increases from 0 to 0.2. It is seen that the values of μ measured with two different discharge tube diameters, fall within the same curve.

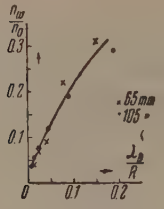

Fig. 1.

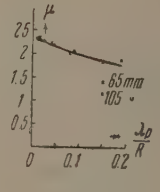

Fig. 2.

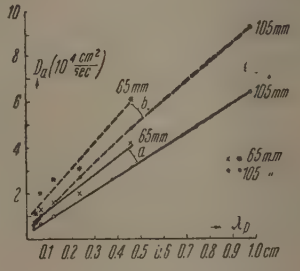

Fig. 3.

The diffusion coefficient D_a was calculated on the basis of equation (11) and presupposing that all the ions generated per unit length of the column recombine at the walls. If the ions are formed directly from atoms in the normale state, the latter assumption is expressed as follows:

$$2\pi R j_p = e \int_0^R z n 2\pi r dr = 2\pi e z n_0 \int_0^R J_0\left(\mu \frac{r}{R}\right) r dr \qquad (12)$$

(j_p — density of the ion current to the walls). From (5′) and (12) we find

$$D_a = \frac{j_p R}{e n_0 \mu J_1(\mu)} \qquad (13)$$

or, expressing $N_e = 2\pi \int_0^R n r dr = \frac{2\pi n_0 R^2 J_1(\mu)}{\mu}$ (number of electrons per cm of column),

$$D_a = \frac{2\pi j_p R^2}{e N_e \mu}. \qquad (14)$$

However, it is quite possible that in the range of pressures and current densities in which we are interested, the ionization of mercury vapour is of a cumulative character (8). Then $z = \beta n_a$, where n_a is concentration of excited atoms. As has been shown by V. A. Fabrikant (9), the space distribution of these atoms departs most from uniformity at low current densities, when the distribution is given by

$$n_a(r) \cong n_{a0} J_0\left(\mu \frac{r}{R}\right).$$

For this case and assuming cumulative ionization, it was established by V. A. Fabrikant that the electron distribution will differ slightly from that of excited atoms. If we consider this case as the opposite to that of direct ionization, we find that the number of ion pairs generated per cm³ per second is equal to $\beta n_{a0} n_{e0} J_0^2 \left(\mu \frac{r}{R}\right)$. Instead of (12) we have

$$2\pi R j_p = 2\pi\varepsilon\beta n_{a0} n_{e0} \int_0^R J_0^2 \left(\mu \frac{r}{R}\right) r dr = \pi\varepsilon\beta n_{e0} n_{a0} R^2 [J_0^2(\mu) + J_1^2(\mu)], \quad (15)$$

and instead of (5′): $\sqrt{\frac{n_{a0}\beta}{D_a}} = \frac{\mu}{R}$.

Combining the last two formulas, we find

$$D_a = \frac{j_p R}{\varepsilon n_{e0} \frac{\mu^2}{2} [J_0^2(\mu) + J_1^2(\mu)]} = \frac{2\pi j_p R^3}{\varepsilon N_e} \frac{2J_0(\mu)}{\mu^3 [J_0^2(\mu) + J_1^2(\mu)]}. \quad (16)$$

In fig. 3 are plotted the values of $D_a = f(\lambda_p)$ calculated from the probe measurement data for both tubes (65 and 105 mm): curves *a*—assuming direct ionization [formula (14)], and curves *b*—assuming cumulative ionization [formula (16)]. These curves give the two limits of possible values of D_a for each value of R. It must be supposed that the true value lies between them—at lower pressures, nearer to curve *a*, and at higher pressures, nearer to curve *b*.

The curves in Fig. 2 and 3 can be used in calculating rates of ionization and deionization in tubes of arbitrary radius, if conditions are such as to satisfy the diffusion laws. In this case, D_a can be evaluated with an error not exceeding 20%.

To what extend such calculation fits in with the facts, can be demonstrated experimentally by means of tests on the rate of deionization. The results of such checking will be given in the next communication.

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Received
7. V. 1939.

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Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1939. Volume XXIII, № 9

PHYSICS

DIFFUSION OF IONS IN A DISCHARGE AND THE INITIAL RATE OF GAS DEIONIZATION. II

By B. L. GRANOVSKY

(Communicated by S. I. Wawilow, Member of the Academy, 6. V. 1939)

1. In the previous paper* there were presented the results of determining the principal quantities necessary in computing the rate of gas deionization under the diffusion regime (²).

These quantities—the ambipolar diffusion coefficient D_a and μ, the root of equation $\frac{n_r}{n_0} = J_0(\mu)$ — were determined in mercury vapour for a cylindrical plasma, at two various diameters. By means of these quantities the deionization «time constant» may be calculated which in the cylindrical case is equal to

$$\tau = \frac{R^2}{D_a \mu^2}. \qquad \text{(II, 1)}$$

During deionization D_a decreases with the electron temperature (¹); therefore τ actually is not constant. However, it may be assumed that at the beginning of the deionization process D_a has the same value as in steady discharge.

Thus, the initial value of $\tau = \tau_0$ can be calculated from the data obtained in steady discharge. Its expression varies according to the manner of ionization of the gas discharge. With direct ionization we find, substituting (I, 14,) into (II, 1) that

$$\tau_0 = \frac{\varepsilon R J_1(\mu) n_{e0}}{\mu j_p} = \frac{\varepsilon N_e}{2\pi R j_p}. \qquad \text{(II, 2)}$$

With cumulative ionization the substitution of (I, 16) into (II, 1) gives:

$$\tau_0 = \frac{\varepsilon R n_{e0}}{2 j_p} [J_0^2(\mu) + J_1^2(\mu)] = \frac{\varepsilon N_e}{2\pi R j_p} \frac{\mu [J_0^2(\mu) + J_1^2(\mu)]}{J_0(\mu)}. \qquad \text{(II, 3)}$$

Note that (II. 2) could be derived in quite an elementary way, were we simply to write down that the decrease in the number of ions (and electrons) in the plasma is equal to the number of ions approaching the walls of the tube. We have per unit length of tube

$$-dN_e = 2\pi R \frac{j_p}{\varepsilon} dt$$

* Referred to, further, as paper I.

hence

$$\tau = -N_e \frac{dt}{dN_e} = \frac{\varepsilon N_e}{2\pi R j_p}.$$

This result shows that formula (II, 2) holds also for low pressures and is not restricted to the diffusion regime only. In this case, however, it expresses the rate of decrease of the total number of ions in the complete plasma cross-section and does not tell anything about the concentration variation in the various parts of the plasma.

2. The initial deionization rates were determined experimentally in mercury vapour by cathode-ray oscillograph recording of the probe current, a description of this method was given previously ([1]).

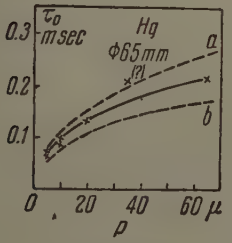

Fig. 1.

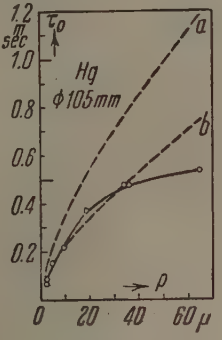

Fig. 2.

In order to obtain a precise reproduction of the process, especially of its most important, initial part, great care was taken in reducing the inertia of the amplifier, i. e. in improving the high-frequency transmission. Control record showed that the amplifier used in our experiments, has a 18 μ sec. time constant; special formulas were derived for calculating the resulting correction factor*. The experiments were carried out in the same discharge tubes in which were measured D_a and μ for the steady discharge plasma. Therefore it was possible to compare the observed values of τ_0 with the values computed from the steady discharge data.

The experimental values of τ_0 measured in a 65 mm diameter tube, are represented in Fig. 1 by the dots and the solid curve; they are determined from the slope of the tangent to the curve $\ln \frac{i'_s}{i'_{s0}} = f(t)$ at its beginning; the correction factor for the amplifier inertia is taken into account. The computed values of τ_0 are represented by the broken line curve: curve *a* is computed by formula (II, 2); curve *b*—by (II, 3); the values of D_a and μ are taken from paper I. Similar data for the 105 mm diameter tube are shown in Fig. 2.

Comparison of the calculated data with those obtained experimentally shows:

a) The absolute values of the initial deionization rates at $pR < 0.2$ mm Hg derived from computations based on the theory of diffusion and on values of D_a determined in steady discharge, are quite near to the actual values.

b) With discharge tubes of a large diameter, the best quantitative agreement with experimental data are obtained with values of D_a calculated on the

* In (2) this factor had not been properly accounted for; thus, the corresponding values of τ_0 for small pR are exaggerated.

assumption of cumulative ionization in steady discharge; with smaller diameters τ_0 and, consequently, D_a have values intermediate between those obtained on both extreme assumptions (only direct, and only cumulative ionization).

c) With the highest values of $pR > 0.25$ cm Hg, the initial deionization rate is greater compared to what it will be when the above assumptions are made.

Hence it follows, in the first instance, that the ambipolar diffusion coefficients D_a in steady discharge and in disintegrating plasma in the beginning of the deionization process have the same values, at least approximately. This means that the beginning of deionization is not accompanied by any abrupt changes in the condition of the plasma. This statement does not in the least contradict the assumption that z is zero under these conditions. Actually z is determined by the existence of fast electrons, disappearing very quickly (1). D_a is determined by the mean electron velocity and varies quite slowly. For instance, if T_e varies from 1.5 eV to 1.2 eV, then D_a changes about 10%, while for direct ionization of Hg z falls 12 times, and for cumulative ionization 3.6 times.

Further, it is worth mentioning that the value of τ_0 at $p > 50\mu$ Hg in a 105 mm diameter tube is lower than these calculated on both assumptions. This discrepancy cannot be explained neither by decrease in the electron energy nor by continuous ionization ($z \neq 0$); these processes can only increase μ (retard the deionization), but not decrease it. The most natural explanation seems to be the appearance of space recombination with the decreasing rate of diffusion towards the walls.

The practical value of the results obtained consists in that the formula (II, 1) and the values of D_a and μ represented on the curves of Fig. 2 and 3 (paper I), may actually be used in the engineering computations of the initial deionization rates in various gas discharge apparatus.

As previously, the experimental data given in the present paper were obtained in collaboration with eng. E. V. Merzloukhova and E. A. Vintilina.

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Received
7. V. 1939.

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Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1939. Volume XXIII, № 9

PHYSICS

EFFECT OF WEAK MAGNETIC FIELD UPON PLASMA

By Ch. M. FATALIEV

(Communicated by S. I. Wawilow, Member of the Academy, 6. V. 1939)

Fundamental works of Langmuir ([1],[2]) on the theory of probes are known to make no allowance for the influence of magnetic field upon probe currents. The conditions for the electrons to be caught by the probe are then essentially altered. An erroneous conclusion which is apt to arise from the disregard of this circumstance is «that the probes in magnetic field fail to detect the increase in electron concentration» ([3]).

M. Steenbeck ([4]) is right while pointing out that our conception of the mechanism underlying the disturbance of the gaseous discharge by the magnetic field is hindered in its further progress by the fact that the probe theory does not involve the factor in question. The respective generalization is made, however, in the works by Spiwak and Reichrudel ([5],[6]). Essential is that the capture of electrons (when magnetic field is available) is bound up with the processes taking place near the probe at a distance d comparable to the mean free path of the electrons.

It is not a difficult matter to interprete these processes geometrically. The general equations of electronic theory may be made the basis to show that an electron near a plane probe is describing a circumference perpendicular to magnetic field H with its center moving uniformly along a line which runs parallel to the probe surface. Take it for granted that the trajectories of the electrons hitting the probe have one common point with, i. e. are tangent to the probe surface, and the condition for an electron to be captured by a plane zone parallel to H will be written as follows

$$2\frac{\varepsilon\varphi}{m}-\frac{\varepsilon^2H^2}{m^2}d^2-2\frac{\varepsilon dH}{m}v+u^2=0. \qquad (1)$$

This expression fits in with that obtained by Spiwak and Reichrudel ([6]). Here ε and m are charge and mass of the electron, v and u are its tangential and normal velocity components, φ is potential of the probe.

2. A discharge tube 50 cm long and 5 cm in diameter was provided with an oxide cathode and a molybdenum anode. Plane molybdenum probe 0.8 cm in diameter available in the tube was devised so as to take any position with respect to the axis of the tube and that of magnetic field. Sufficiently homogeneous magnetic field applied to the tube from outside was generated by a solenoid 80 cm long and 23 cm in diameter.

Fig. 1 presents semilogarithmic characteristics of a plane probe placed on the axis of the tube in argon discharge with a pressure of 0.9 mm Hg and

a discharge current equal to 100 mA. The characteristics on the left and the right hand side relate to a probe whose plane is parallel or perpendicular to H respectively (the corresponding values of disturbing magnetic fields are expressed in gausses in the drawing). The characteristics of a plane probe perpendicular to H were not liable to change if the magnetic field would not disturb the plasma as a whole. The curves in Fig. 1 show, however, that the plasma being disturbed by the field, the characteristics shift up the axis of ordinates. And the more so, the higher is the magnetic field. Such a shift should be interpreted in terms of increased concentration of electrons in the point in which the probe is situated.

The increase of electron concentration with H is brought out in Fig. 2 for a case of neon discharge at a pressure of 0.2 mm Hg. The characteristics show that under conditions of our experiment the space potential varies within the error involved in the method of determination applied. More proper conditions, however, would reveal that the space potentials tend to become somewhat negative (with respect to anode); this is substantiated by certain increase in the total voltage of the tube observed on switching in the magnetic field. The characteristics of a probe parallel to H disclose changes peculiar to probe currents in a magnetic field: near and above the space potential the field is seen to reduce the electronic currents coming to the probe from the plasma (d-effect). It is worthy of note that the value of the discharge current is also affected (albeit inconsiderably) by the presence of a longitudinal field. In order to be convinced that the factor decreasing the probe currents is here the cycloidal motion of electrons rather than the decrease of discharge current it was necessary to maintain the latter at a constant value. The fact that below the space potential and with high negative probe potentials there is taking place an increase of currents in the probe chain, should be taken to mean that the motion of fast electrons is but little affected by the d-effect and that the latter is moreover overcome in this region of the characteristic by a much more potent factor, the disturbance of plasma.

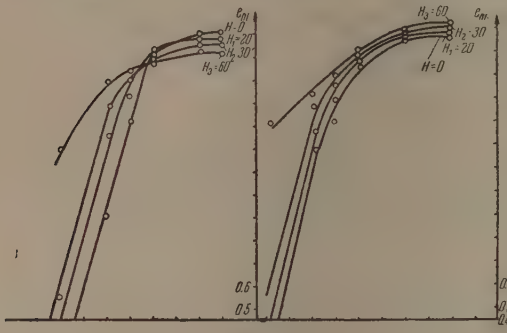

Fig. 1.

All the measurements made with weak fields give an obviously linear run of the probe characteristic with the plane perpendicular to H. The distortions in the upper part of the characteristics obtained for a probe parallel to H may be attributed to the cycloidal motion near the probes. These results furnish reliable evidence as to the existence of Maxwell's velocity distribution among electrons in the plasma disturbed by a weak magnetic field. Therefore, if the probe is directed to eliminate the d-effect, the electron temperature is determined in the usual way from the tangent of the inclination angle([1]).

In the case of noble gases (neon, argon) another effect of the field is markedly to reduce the inclination of the characteristic for a probe perpendicular to H. This is probably because the electron temperature in magnetic field has increased. The conclusion recently drawn by Fabrikant and Rochlin ([7]) from optical data is that magnetic field lowers the electron temperature.

Their experiment has been run with mercury arc under low pressure and strong field conditions. Within a certain range of $\frac{E}{p}$ values (E—gradient in the column, p—pressure) there shall be a linear relation between E and electron temperature, but beyond this range the temperature varies slower than does the gradient. The evidence furnished by Spiwak and Reichrudel[8] is that the gradient grows with H in the case of argon and neon, while it goes down as H goes up in the case of mercury vapours. Naturally enough, one should expect the temperatur to increase in the first case and to drop in the second.

3. With a plane probe the quantitative estimation of the two principal effects of magnetic field separately—the disturbance of the whole of the plasma and the d-effect becomes possible. If $i_\parallel$ and $i'_\parallel$ are currents directed toward a probe which is parallel to H, before and after the magnetic field has been applied, $i_\perp$ and $i'_\perp$ are the respective values for a probe perpendicular to H, i_e and i_p are electronic and ionic components of non-disturbed current, i'_e is electronic current which will arrive at a probe parallel to H if H is considered acting nowhere except in the vicinity of the probe, then we may write:

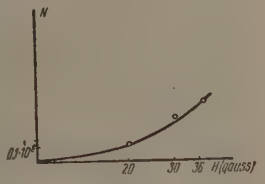

Fig. 2.

$$i'_e = i_e s$$
$$i'_\parallel = s'(i_e s - i_p) \qquad (2)$$

and for the upper part of the characteristic with $i_e \gg i_p$

$$s = \frac{i_\perp}{i'_\perp}\,\frac{i'_\parallel}{i_\parallel}$$
$$s' = \frac{i'_\perp}{i_\perp} \qquad (3)$$

where $s < 1$ implies a decrease in probe currents due to d-effect, while $s' > 1$ means that the whole of the plasma is disturbed by magnetic field (change in concentration and temperature of electrons). From the probe theory[6] we obtain for s the expressions:

if $\eta > 0$, $s = \frac{1}{2}\left\{1 \pm \Phi\left(\pm\frac{\eta}{2\xi d} \mp \frac{\xi d}{2}\right) + e^{\eta}\left[1 - \Phi\left(\frac{\eta}{2\xi d} + \frac{\xi d}{2}\right)\right]\right\}$

if $-\eta \gtrsim \xi^2 d^2$,

$$\eta < 0,\ s = \frac{1}{2}\left\{1 - \Phi\left(\frac{\xi d}{2} - \frac{\eta}{2\xi d}\right) + e^{\eta}\left[1 \pm \Phi\left(\mp\frac{\eta}{2\xi d} \mp \frac{\xi d}{2}\right)\right]\right\} e^{-\eta}$$

if $-\eta \gtrsim \xi^2 d^2$,

where $\eta = \frac{\xi\varphi}{kT}$, $\xi = \frac{\varepsilon H}{\sqrt{2mkT}}$, $\Phi(x) = \frac{2}{\sqrt{\pi}}\int_0^x e^{-y^2} dy$. (4)

d may be determined by joint solution of (3) and (4). It takes a value which is near to the mean free path of the electrons in the plasma. This serves to check the experimental method of separate consideration of effects produced by magnetic field as well as to verify the chief premise of the probe theory

with reference to the case concerned. Besides. the dependence of s upon φ as obtained from empirical formulae (3) does not differ from what it appears to be in theoretical relation (4).

In order to properly determine the electron concentration N_H in magnetic field, use must be made of the relation

$$i_{0\parallel}' = sN_H e\sqrt{\frac{kT_H}{2\pi m}}. \qquad (5)$$

If the characteristics of a probe parallel to H are treated according to Langmuir (i. e. d-effect is ignored), there will be such a case of discharge (argon, p=0.9 mm, discharge current—100 mA) where the error in N_H amounts to the order of 20—30% even for a weak field. By providing respective conditions the order of magnitude assumed by the error may be made higher still.

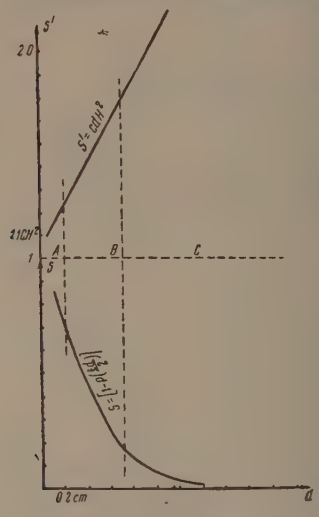

Fig. 3.

The changes experienced by the characteristics of the probe in a magnetic field and characterized by the values of s and s' depend upon the mean free path of the electrons in the plasma. Within a certain range of pressures which varies according to the gas concerned (equal to 0.5 mm Hg in the case of neon), the characteristics bring out the d-effect with sufficient clearness and, besides, they display unmistakable signs which point to the disturbed condition of the plasma. When we proceed from this range in the direction of higher pressures, we find, as it was to be anticipated indeed, that d-effect and the disturbance of plasma both keep decreasing (s and s' both tend to unity). If, however, we will take the direction of lowering pressures, without changing the value of H, we will observe a strong increase in the disturbance of plasma (s' notably greater than unity), on the one hand, and a nearly vanished response of the probe to d-effect, on the other ($s \approx 1$). In this region of pressures the characteristics of the probe in magnetic field may practically be treated according to Langmuir without much increasing the error. But outside of that narrow region the use of the classical method involves great inaccuracy. To interprete such a character of the relation between the effects of magnetic field and the pressure is not a difficult matter if we, first, take into account that there are some reasons ([9],[10]) for considering the disturbance of plasma by magnetic field (and with it the value s') as proportional to dH^2 and, secondly, trace the variation of s with d in (4) (with space potential $s=1-\Phi\left(\frac{\xi d}{2}\right)$ and is to a certain extent saturated at high values of d) (Fig. 3).

One is tempted to suppose that a longitudinal magnetic field concentrates the discharge about the tube axis at the expense of decreasing density of electrons near the walls. Experiments show, however, that within the studied interval of pressures (from tenth of a mm to 1 mm Hg) the electron concentration in a neon or an argon discharge grows all over the seation of plasma (up to such distances from the wall as are comparable to the mean free path of electrons). Thus, in the case of neon discharge, pressure—0.2 mm

Hg, discharge current—150 mA and $H=36$ gausses, the values of electron concentrations are 15—20% higher than they are in the absence of magnetic field. It is therefore the average electron concentration which is increased in the plasma by the action of a longitudinal magnetic field.

4. The latter tells also upon the conditions prevailing on the walls. From the measurements made with a plane probe when in close contact to the wall we may conclude that in a magnetic field the potential of the wall becomes more negative (in argon and neon) with respect to the anode. With a given H the change in the wall potential is the better expressed the lower is p and to a certain extent is also dependent upon the value of the discharge current. This variation of the wall potential is related to the above-mentioned increase in the total voltage on the tube as well as to the potential change within the volume of the tube. If the ionic currents on to the wall are determined by extrapolating the volt amper characteristic to the potential at which the total current shared to the probe is zero, it will become evident that the effect of applied magnetic field is to reduce the ionic (and the electronic) currents directed to the walls. Several different effects of magnetic field must combine to produce a decrease in those currents. The first factor is the decreased coefficient of diffusion of charged particles toward the walls.

Then seem to come processes which take place at the last free path of electrons near the wall. Part of the electrons moving from the plasma to the wall and subjected within this interval to the electric field of the wall, on the one hand, and to its perpendicular magnetic field, on the other, will turn back to the plasma, and it is the two velocity component of the electron (normal and tangential) which determines whether it will or will not hit the wall.

It is a pleasure to acknowledge advice and criticism received from Prof. Spiwak, by whom the author was led in this work.

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Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1939. Volume XXIII, № 9

PHYSIQUE

DÉSINTÉGRATION DE L'URANIUM PAR LES NEUTRONS

Par N. PERFILOV.

(Présenté par V. G. Khlopin, de l' Académie le 4. V. 1939)

Dans une chambre de Wilson à pression réduite ont été effectuées des recherches sur la désintégration de l'uranium par les neutrons. Une pellicule de celluloïde d'épaisseur de 10 microns et de surface de 12 cm^2, fixée sur un filet de laiton aux troux de 0.25 cm^2 et contenant U_3O_8 distribué par tout le volume, a été placée dans la chambre et irradiée par des

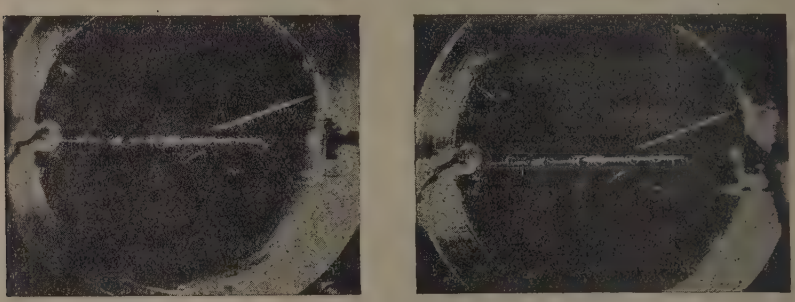

Fig. 1.

neutrons produits par une source de Rn+Re dont l'intensité était de 1000 mCu. A la pression d'air et de vapeur d'alcool à l'intérieur de la chambre égale à 25 cm de Hg, on a obtenu 1000 stéréophotographies.

Ces clichés permettent de faire quelques conclusions concernant la charge et la masse des noyaux de recul de l'uranium. Tandis que sur de certains clichés les traces des noyaux de recul se distinguent nettement par l'ionisation des particules α, sur d'autres on ne peut pas remarquer de grande différence dans l'ionisation produite. De l'origine de ces traces on ne peut juger que d'après le fait qu'elles partent d'un point de la plaque dans les sens opposés. Ces particules appartiennent-elles vraiment à la désintégration de l'uranium sous l'action des neutrons, ou bien leur apparition est due à une coïncidence accidentelle des directions et du point de départ des particules α ordinaires—on ne pourrait le dire qu'après l'examen définitif de tous les matériaux expérimentaux obtenus.

Sur la fig. 1 on peut voir que la trace d'une lourde particule est beaucoup (au moins 6 fois) plus intense que celles des particules α.

Cela nous permet de supposer que la charge du noyau de recul doit être grande. Sur le cliché on ne voit que la trace d'un débris qui se termine pro-

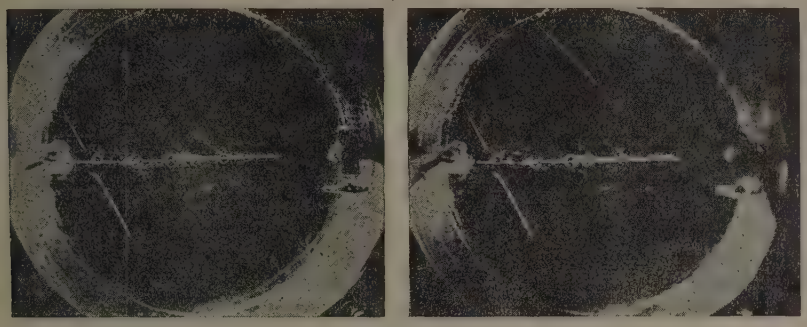

Fig. 2.

bablement à l'intérieur de la plaque à cause du grand angle d'inclinaison, ou aboutit à la carcasse du filet sur lequel est fixée la pellicule. Sur la fig. 2

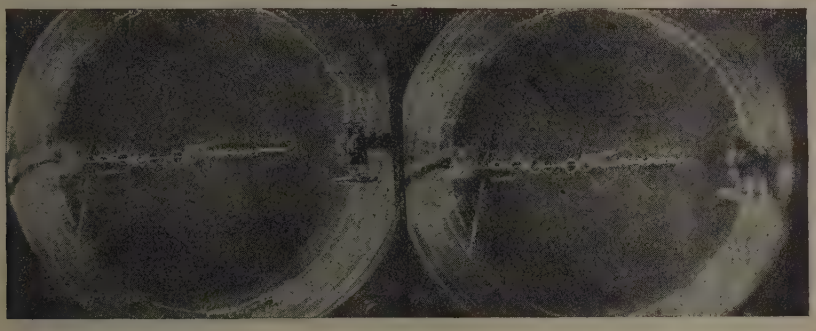

Fig. 3.

on voit les traces de deux noyaux de recul. Les deux traces se distinguent des traces des particules α par leur densité, mais la différence est moins pronon-

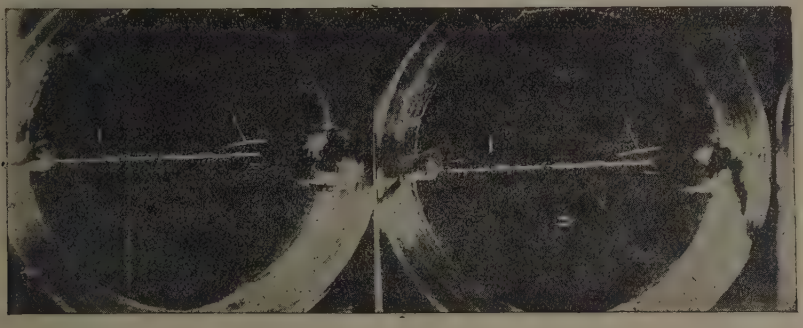

Fig. 4.

cée que celle de la trace de la particule sur la fig. 1. Il y a des clichés oú la densité des traces des noyaux de recul est différente des deux côtés de la

plaque. La cause réside peut être dans le fait qu'au partage de l'uranium les différents débris peuvent acquérir des charges différentes. Le cliché 3 permet de faire cette supposition. D'un côté de la plaque la particule produit une ionisation de gaz plus grande que celle qui est causée par la radiation α, et le deuxième débris de l'uranium, faisant son parcours dans le sens opposé, ne se distingue pas par son ionisation d'une particule à double charge.

Sur la fig. 4 est représentée une des fourchettes obtenues. On ne peut l'attribuer au choc élastique des deux particules parce que l'angle formé par la direction initiale de la particule partant de la plaque et la direction de la trace déviante dépasse 90°. Peut être que c'est le partage du noyau du reste de l'uranium qui est représenté par cette fourchette.

La longueur maximum du parcours des noyaux de recul par rapport à l'air normal est d'environ 3 cm. Actuellement les expériences continuent et une description détaillée des resultats sera donnée plus tard.

Il ne me reste qu' à remercier M. le professeur L. Myssowsky de ses conseils et des instructions précieuses faites au cours de ce travail.

Institut de Radium.
Académie des Sciences de l'URSS.

Manuscrit reçu
le 9. V. 1939.

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Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1939. Volume XXIII, № 9

PHYSICS

ON QUANTUM ELECTRODYNAMICS. II

By V. L. GINSBURG

(Communicated by V. A. Fock, Member of the Academy, 29. III. 1939)

In order to inquire into the nature of the difficulties met with in quantum electrodynamics it is highly important to throw light on the question as to which of the divergent expressions available in theory are endowed with classical analogies and which of them bear a purely quantum character. It is this very question relating to discrepancies connected with a transverse magnetic field which fails to be altogether clear. The fact is that the calculation of transverse divergences is usually conducted on such assumptions as make it impossible to effect a comparison of quantum results with the customary classical ones. Namely, in these accounts the reaction of emission to the electron is not neglected, while the classical expressions for the energy of the interaction of the electron with its own field, as well as the energy of the entrained transverse field always refer to the stationary case (it is a uniformly moving electron we are concerned with throughout this work). If in classical theory, in analogy with the quantum statement of the problem, the assumption is made that with $t=0$ there exists a uniformly moving electron, while the energy of the transverse field is equal to zero, then, upon adiabatic inclusion of the interaction between the electron and the field, the electron will, as has already been shown in communication I ([1]), radiate its own entrained stationary field only provided the reaction of emission be left out of account. The physical import of this result is perfectly clear. With due account of the reaction taken it will be obvious that, as emission progresses, the electron is liable to slow down. Upon slowing down it begins to radiate otherwise, etc. Finally, at the end of the process of inclusion, the electron will be endowed not with initial impulse $\vec{p}$, but with some other, lesser impulse, while the energy emitted by it will, certainly, not be equal to the energy of the field entrained in the stationary case by an electron with initial impulse.

Besides the energy of the entrained field in H (I, 14) there makes its appearance also the energy of the interaction of the electron with its own field which in sum with the first is equal to the energy of disturbance E_p. With this:

$$E_p = -\frac{4\pi e^2}{m^2}\sum_{\lambda}\frac{(\vec{p}\vec{e}_\lambda)}{\nu_\lambda^2} = -\frac{4e^2}{3\pi c^3}\left(\frac{\vec{p}^2}{2m^2}\right)\int_0^{\nu_{max}} d\nu, \qquad (1)$$

if the spectrum be broken off at frequency ν_{max}. Thus, for a point electron E_p diverges as $\int_0^{\nu_{max}} d\nu$ with $\nu_{max} \to \infty$. Let us recall that in the respective calculation we also neglected term $\frac{e^2}{2mc^2}\vec{A}^2(r)$ in (I, 1). It is particularly important that, as shown in I, precisely the same result is obtainable in the quantum theory. Hence we conclude that the discrepancy due to term $-\frac{e}{mc}\vec{p}\vec{A}(r)$ in (I, 1) bears a classical character.

In obtaining expression (1) we did not make use of the theory of disturbance, yet it is of interest to note that the second approximation of this theory leads to the same results. The second approximation for energy of disturbance depending upon terms $-\frac{e}{mc}\vec{p}\vec{A}$ and $\frac{e^2}{2mc^2}\vec{A}^2 b$ in (I, 1) has the following appearance:

$$E'' = \sum_{m \neq 0} \frac{\left|\left(m\left|-\frac{e}{mc}\vec{p}\vec{A}\right|0\right)\right|^2}{E_0 - E_m} + \left(0\left|\frac{e^2}{2mc}\vec{A}^2\right|0\right) =$$

$$= E_1'' + \left(0\left|\frac{e^2}{2mc}\vec{A}^2\right|0\right). \qquad (2)$$

Accurate computation of E_1'' leads to a complex expression as suggested by Waller (2). Let us remark that Waller's expression yielding a diverging result even when $\vec{p} = 0$, is erroneous for this particular value of $\vec{p}$. It will be easily seen that $E_1'' = 0$, if $\vec{p} = 0$. The obtained result which is true also for (1), according to which E_1'' [or E_p in case (1), with $\nu_{max} \to \infty$] will be equal to infinity when $\vec{p} \neq 0$ and $E_1'' = E_p = 0$ when $\vec{p} = 0$, may be accounted for in the following way. If the electron is quiescent, then its transverse field * is in general equal to zero (the zero conditions of oscillators of field available in the quantum case do not alter matters, since only the mean value of the field is of importance) and consequently $E_1'' = E_p = 0$; but in case if the point electron moves, then with any small value of $\vec{p}$ the energy of its transverse field is infinite.

Neglecting the reaction means that we regard the energy and the impulse of the electron in conditions 0 and m as equal, whence $E_0 - E_m = -\hbar\nu_m$. For E_1'' expression (1) is thereby obtained. A change in the energy of the transverse field, upon the inclusion of disturbance, will in the second approximation and with omissions of the reaction be equal to:

$$U = \int \Psi_1^* H_{tr} \Psi_1 \, dq - \int \Psi_0^* H_{tr} \Psi_0 = -E_1'' = -E_p \, *. \qquad (3)$$

In (3) Ψ_1 and Ψ_0 are respectively functions of the first and zero approximations, H_{tr} is determined according to (I, 5) and q denotes the sum total of the co-ordinates of the particle and the field. For the change in the energy of interaction we obtain:

$$V = \int \Psi_1^* \left(-\frac{e}{mc}\vec{p}\vec{A}\right) \Psi_1 dq - \int \Psi_0^* \left(-\frac{e}{mc}\vec{p}\vec{A}\right) \Psi_0 \, dq = 2E_1'' = 2E_p. \qquad (4)$$

* This calculation reproduces virtually the results of Smirnov, to which reference was made in I.

Expressions (3) and (4) are in good agreement with (I, 14). Calculation of the second term in (2) leads to the following result irrespective whether or not we neglected the reactions:

$$E_2'' = \left(0 \left| \frac{e^2}{2mc^2} \vec{A}^2 \right| 0\right) = \frac{e^2\hbar}{\pi mc^3} \int_0^{\nu_{max}} \nu d\nu. \qquad (5)$$

E_2'' differs from zero since $(0|q_{\lambda i}^2|0) = \frac{\hbar}{2V_\lambda} \neq 0$, i. e. due to fluctuations of the oscillators of the field under zero condition. Consequently, divergence from term $\frac{e^2}{2mc^2}\vec{A}^2$ bears a purely quantum character.

Rosenfeld ([3]) believed that this divergence could be got rid of in the same way as for zero energy in H_{tr}. Rosenfeld's opinion is, however, based on a transformation which is erroneous, though it is frequently resorted to in quantum electrodynamics. As will be seen from (I, 4), only co-ordinates of field $q_{\lambda i}$ enter the expression for $\vec{A}$, while both $q_{\lambda i}$ and $p_{\lambda i}$ enter that for H_{tr}. If we deal with a pure emission field, then $q_{\lambda i}$ is subordinate to a homogeneous oscillatory equation and it will be permissible to express $p_{\lambda i} = q_{\lambda i}$ simply by $q_{\lambda i}$, i. e. it is possible to eliminate impulses from H_{tr}. After this, with the help of canonic transformation, H_{tr} again assumes the aspect of the sum of the energies of oscillators [for details see the work by Heitler ([4])], but in this case there also appear in the expression for $\vec{A}$ impulses and co-ordinates of field. In the general case when there are particles in the field, $q_{\lambda i}$'s will conform to equations (I, 6) and their dependence upon time will be unknown. Therefore the aforesaid transformation with the availability of particles is inadmissible and we fail to see any way which might lead to the simultaneous presence in $\vec{A}$ of both co-ordinates and impulses. Inasmuch as the way of elimination of the divergence from term $c\vec{A}^2$, as suggested by Rosenfeld, is based precisely upon a permutation of the non-commutative factors in $\vec{A}^2$, we are bound to recognize it as erroneous. It is of course possible, while making use of the connexion between $(0|q_{\lambda i}^2|0)$ and $(0|p_{\lambda i}^2|0)$, to introduce impulses into integral expressions, and in the subsequent to «eliminate» the divergence of these expressions, yet we do not believe that such elimination is in any way preferable to simply crossing out troublesome terms.

In the case of an electron conforming to Pauli's equation, provided the reaction and the term with cA^2 in the operator of energy be omitted, it is possible to obtain the following result (I):

Here, apart from the already mentioned terms, the energy of disturbance is supplemented by an expression equal to the sum of energy of interaction of the magnetic moment of the electron with its own magnetic field and the energy of the field of this moment. The mentioned energy of disturbance is equal to:

$$E_\mu = -4\pi \sum_\lambda \mu_0^2 \left(\vec{\sigma}\left[\frac{\vec{k}_\lambda}{|\vec{k}_\lambda|}\vec{e}_\lambda\right]\right) = -\frac{e^2\hbar^2}{2\pi m^2 c^5} \int_0^{\nu_{max}} \nu^2 d\nu. \qquad (6)$$

Here $\mu_0 = \frac{\hbar e}{2mc}$, and $\vec{\sigma}$ is Pauli's spinal vector ($\sigma_{x_i}^2 = 1$; $\sigma_{x_i}\sigma_{x_j} + \sigma_{x_j}\sigma_x = 0$

when $i \neq j$. The same result is arrived at in the classical theory of the magnetic electron if it is assumed that $\vec{\mu}^2 = \frac{3\hbar^2 e^2}{4m^2c^2}$, i. e. if it is admitted that the magnetic moment of the electron is equal to the value adopted in the quantum theory. Hence the classical nature of (6) is brought out most clearly. Expression (6) is obtainable also in the second approximation of the theory of disturbance, provided the reaction is neglected. Moreover, it can be easily shown that any change in the energy of the transverse field after inclusion of the disturbance equalling the energy of the magnetic field produced by moment $\vec{\mu}$ is equal to $-E_\mu$. The change in the energy of interaction equals $2E_\mu$.

For the electron described by the equation of Dirac, exact expressions for the second approximation of the disturbance energy (disturbing term $-e\vec{\alpha}\vec{A}$) have been obtained by Waller ([2]) and Weisskopf ([3]). In the case when the electron is quiescent (to which case we confine ourselves) below this energy is exactly equal to E_2'' [see (S)].

If in the second approximation we break up the disturbance energy E'' into a part associated with transitions of the electron to states with positive energy $E''_{w_m>0}$ and a part associated with transitions to states with negative energy $E''_{w_m<0}$, the exact expressions for $E''_{w_m>0}$ and $E_{w_m<0}$ will have a very cumbersome appearance. In this case, of course, $E''_{w_m>0} + E''_{w_m<0} = E'' = E_2''$. But if we consider the energy of the electron in its terminal state w_m as equal to the energy of the initial state $w_0 = mc^2$ in calculating $E''_{w>0}$ and as equal to $-mc^2$ in calculating $E''_{w<0}$, we obtain:

$$E''_{w_m=-mc^2} = -\frac{e^2\hbar^2}{2\pi m^2c^5}\int_0^{\nu_{\max}} \frac{\nu^2 d\nu}{1-\left(\frac{\hbar\nu}{mc^2}\right)} \quad * \tag{7}$$

$$E''_{w_m=-mc^2} = \frac{e^2\hbar}{\pi mc^3}\int_0^{\nu_{\max}} \nu d\nu + \frac{e^2\hbar^2}{2\pi m^2c^5}\int_0^{\nu_{\max}} \frac{\nu^2 d\nu}{1-\left(\frac{\hbar\nu}{mc^2}\right)^2} -$$

$$+\frac{e^2\hbar}{\pi mc^3}\int_0^{\nu_{\max}} \frac{\left(\frac{\hbar\nu}{mc^2}\right)^2 \nu d\nu}{2+\frac{\hbar\nu}{mc^2}-\left(\frac{\hbar\nu}{mc^2}\right)^2}. \tag{8}$$

For a quiescent electron neglecting the reaction we should expect to obtain for E'' expression $E_\mu + E_2''$ following from Pauli's theory. The point is, however, that in Dirac's theory the reaction evidently cannot be altogether neglected, i. e. we cannot consider $w_m = w_0$ and $p_m = p_0$, provided we do not simultaneously omit from our consideration the phenomena connected with the spin. Indeed, a description of the magnetic properties of electron and spin in Dirac's theory is not so much made manifest by the availability of special additional terms in the equations,

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* The division of E'' into $E''_{w_m>0}$ and $E''_{w_m<0}$ was already applied by Beck ([6]). This author pointed out that under the condition $\frac{\hbar\nu}{mc^2} \ll 1$ the obtained expression for $E''_{w_k=mc^2}$ is equal to the energy of the field of the magnetic moment, and $E''_{w_k<0}$ diverges as $\int_0^{\nu_{\max}} \nu d\nu$ when $\nu_{\max} \to \infty$. T..e absence in Beck's paper of any discussion of the character and the results of the approximations made by him deprive us of the possibility of making a reference to his paper.

but rather, formally speaking, by the dependences of the amplitudes of wave functions upon $\vec{p}$. Yet, if in our problem we consider that $\vec{p}_m = \vec{p}_0 = 0$ instead of $\vec{p}_m = \pm \hbar \vec{k}_m$, we virtually discard this dependence and indeed obtain $E''_{w_m>0} = 0$.

Expression (7) is obtained on the condition that $w_m = w_0 = mc^2$, while $\vec{p}_m = \pm \hbar \vec{k}_m \neq \vec{p}_0$. For this reason (7) is comparable with expression (6) of Pauli's theory, obtained under the condition of completely neglecting the reaction, but only in the region of the frequencies where $\vec{p}_m$ is small as compared with mc, i. e. if $\frac{\hbar\nu}{mc^2} \ll 1$; (6) and (7) are, in point of fact, found to be identical.

Transitions to states with negative energy can nowise be regarded as occurring without reaction, since in this case $w_0 - w_m = 2mc^2$, i. e. Δw is very great. For this reason alone it will be difficult to seek for analogies for expression (8) in the classical theory, even if no attention is paid to the fact that $w_m < 0$.

Let us also remark that the divergent expression E''_2 appearing in the theories of Schrödinger and Pauli from term $\frac{e^2}{2mc^2}\vec{A}^2$ in the case of Dirac's theory enters $E''_{w_m<0}$, i. e. is connected with transitions into negative conditions. This result it associated with the well-known circumstance pointed out by I. E. Tamm [7] that dispersion on a quiescent electron which, according to Schrödinger's theory, depends upon the term with $\vec{A}^2$ will in Dirac's theory be brought about by transitions to intermediate conditions with negative energy. The changes which can be brought in by the theory of «holes» [8] have been left out of consideration.

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Received
17. IV. 1939.

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Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1939. Volume XXIII, № 9

TECHNICAL PHYSICS

SELF-INDUCTION AND CAPACITY OF A HERTZ VIBRATOR AND ITS EQUIVALENCY TO A CLOSED OSCILLATORY CIRCUIT

By A. G. ARENBERG

(Communicated by M. W. Shoulejkin, Member of the Academy 17. IV. 1938)

I. In our preceding communication* devoted to an investigation of the radiation resistance of a Hertz vibrator we have shown that for computing this resistance with the help of the method of «induced electromotive forces» it is only necessary to take account of the «active» («watt») components of these e. m. f.'s ($\mathcal{E}_{vz}^{(A)}$ and $\mathcal{E}_{sv}^{(A)}$), for whose amplitudes the respective formulae were obtained (I, 12).

However, from the general expressions for induced e. m. f.'s (I, 10) it follows that, along with these active components of e. m. f.'s proportional to sin ωt, there also exist «reactance» («wattless») components of e. m. f.'s proportional to cos ωt. Knowledge of these components permits of determining the self-induction and capacity of an oscillating vibrator and to reduce it to the terms of an equivalent closed oscillatory circuit.

2. Indeed, if we neglect the magnetic field for the currents running in rhe terminal balls of a vibrator (Fig. 1), we have, approximately for the eactance component of e. m. f. determined by vector potential A:

$$\mathcal{E}_{vz}^{(R)} = -\frac{\omega I_0 \cos \omega t}{c^2} \int_0^D \int_0^D \frac{\cos k \sqrt{\rho^2 + (\xi - z)^2}}{\sqrt{\rho^2 + (\xi - z)^2}} d\xi\, dz \cong$$

$$\cong -\frac{\omega I_0 \cos \omega t}{c^2} \left[2D \left(\ln \frac{D}{\rho} - 0.31 \right) - \frac{k^2 D^3}{3 \cdot 2!} + \frac{k^4 D^5}{10 \cdot 4!} - \frac{k^6 D^7}{21 \cdot 6!} + \dots \right] \qquad (1)$$

where D is the length of the connecting rod of the vibrator, and ρ is the radius of this rod.

Expression (1) shows that the coefficient of self-induction of an oscillatory vibrator (expressed in units of the CGSM system) is determined by the series

$$L = L_0 - \frac{k^2 D^3}{3 \cdot 2!} + \frac{k^4 D^5}{10 \cdot 4!} - \frac{k^6 D^7}{21 \cdot 6!} + \dots, \qquad (2)$$

where

$$L_0 = 2D \left(\ln \frac{D}{\rho} - 0.31 \right).$$

* Cf. Comptes de l'Académie des Sciences de l'URSS, XXIII, № 3 (1939). The notation used in that paper, referred to further by I, is retained here.

As will be easily seen, the first term of this series (L_0) is an expression for the self-induction coefficient of a rectilinear cylindrical conductor with a high-frequency current, as is ordinarly obtained from the well-known formula of Neumann *. With the subsequent terms of the series allowance is made for changes in the self-induction of a vibrator on its radiation, which may be quite appreciable **.

3. In analogy with this, assuming approximately that all the charges of the vibrator are concentrated in its balls, we have for the reactance component of e. m. f. conditioned by scalar potential φ:

$$\mathcal{E}_{sz}^{(R)} = \frac{I_0 \cos \omega t}{\omega} \left| \frac{\cos k(l-z)}{l-z} - \frac{\cos kz}{z} \right|_{l}^{l-\delta} =$$

$$= \frac{2I_0 \cos \omega t}{\omega} \left\{ \frac{l-2\delta}{(l-\delta)\delta} + \frac{k^2(l-2\delta)}{2!} - \right.$$

$$\left. - \frac{k^4}{4!}\left[l-\delta)^3 - \delta^3 \right] + \frac{k^6}{6!}\left[(l-\delta)^5 - \delta^5 \right] - \ldots \right\} \qquad (3)$$

The obtained result shows that the capacity of an oscillatory vibrator (expressed in units of the CGSM system) is determined by the series

$$\frac{1}{C} = \frac{1}{C_0} + 2\left\{ \frac{k^2 D}{2!} + \frac{k^4}{4!}[(D+\delta)^3 - \delta^3] + \frac{k^6}{6!}[(D+\delta)^5 - \delta^5] - \ldots \right\} \qquad (4)$$

where

$$C_0 = \frac{(D+\delta)\delta}{2D}.$$

Comparing the expression for C_0 with the formula for capacity of two balls obtained by R. Dean ([8]) ***, it is easy to see that quantity C_0 determines the electrostatic capacity of a system made up of two balls with radius r, whose electrical centres have heen shifted by $r-\delta$ relatively to their geometrical centres separated from one another by distance l (Fig. 1). The other terms of this series take account of the changes in the capacity of the vibrator with its radiation.

It is obvious that with a sufficiently great length of the vibrator ($D \gg r$), the electrical charges of the balls may be assumed to lie in their geometrical centres ($\delta \to r$) and the electrostatic capacity of the system will be determined as $C_0 = \frac{C_1}{2}$ where C_1 is the own capacity of each of the balls ****.

Thus, the full «reaction of radiation» of the vibrator consists in there being set up in it two electromotive forces:

$$\mathcal{E}_{vz} = \mathcal{E}_{vz}^{(A)} + \mathcal{E}_{vz}^{(R)} \quad \text{and} \quad \mathcal{E}_{sz} = \mathcal{E}_{sz}^{(A)} + \mathcal{E}_{sz}^{(R)}, \qquad (6)$$

one of which ($\mathcal{E}_{vz}$) is determined by vector potential $\bar{A}$, while the other one ($\mathcal{E}_{sz}$) by scalar potential φ. Here the most important circumstance is that in

* The deduction of this formula for L_0 and its comparison with the formulae which H. Hertz used ([1]) are stated in H. Poincaré's book ([2]). Cf. also ([3]—[6]).

** In this connexion it should be noted that the change in the self-induction coefficient for a closed circuit in course of its radiation was calculated by L. Brillouin ([7]).

*** In the stated work Dean takes his starting point in the possibility of substituting spherical surfaces for equipotential surfaces of charges located at 2 points. Cf. also the works of W. Thomson ([9]), Hankel ([10]), D. Bobyleff ([11]), Kirchoff ([12]) and others on the distribution of electricity on 2 balls.

**** A similar expression for the capacity of a vibrator is referred to in the foot-note to Hertz's article published in 1891 under the title «On very rapid electrical vibrations» ([13]). It will be recalled that H. Poincaré ([14]) was the first to point to the necessity of taking account of C_0 in a similar way.

each of these e. m. f.'s an active component is to be found, and this seems to constitute the basic distinction of radiating electromagnetic systems from non-radiating ones which fail to contain active components.

The above-mentioned permits us to plot a vector diagram of a vibrator, radiating non-damping electromagnetic waves. In constructing this diagram it should be remembered that the e. m. f. induced in the vibrator by the feeding source («foreign e. m. f.») is connected with the geometrical sum of all the induced e. m. f. 's by means of the necessary relationship $\mathcal{E}_f = -\mathcal{E}_z$.

4. Let us now consider the problem of the equivalency of a Hertz vibrator to a closed oscillatory circuit which was first considered by R. Rudenberg [15] (with reference to the case of a vibrator excited by passing electromotive waves) and later on discussed by A. A. Petrovsky [16].

On the basis of the results obtained by H. Lorentz [17], M. Abraham [18] and M. Planck [19], Rudenberg compared an oscillatory vibrator to a closed vibratory circuit into whose energy equation he introduced an additional term to take care of radiation.

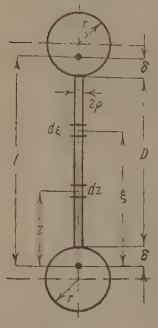

Fig. 1.

On the basis of the classical expression for radiated power (P_Σ) given by Hertz [20], Rudenberg adopted this additional term (a counterpart of the term for «radiant brake-effect» due to harmonical vibrations of an elastically bound electron) in the form $P_\Sigma = -S\dot{q}\ddot{\ddot{q}}$ (here $S = \frac{2}{3}\frac{l^2}{c^3}$ is «coefficient of radiation» depending only on the size of the vibrator, and q is charge of the vibrator). Further, on having made an equation for the e. m. f. of an equivalent vibratory circuit, Rudenberg determined its resonance frequency ω_0 and obtained an expression for radiation resistance ($R_\Sigma = \omega^2 S$) corresponding to formula (I, 6).

It is easy to see, however, that a similar reduction of radiating vibrator to the terms of an equivalent closed oscillatory circuit appears to be still more artificial and formal than the very deduction of the formula of «radiant brake-effect» mentioned above*.

In fact, the basis of Rudenberg's reasoning referred to here is to consider only the «active» power (P_Σ) radiated by the vibrator and determined by means of integrating the mean flux of the radial component of Pointing's vector (S_r) over a sphere comprising the whole vibrator. It is, however, obvious that a strictly performed reduction of the vibrator to an equivalent closed oscillatory circuit requires a much fuller consideration of electromagnetic processes connected with the formation of both active and reactance fluxes of energy occurring in the immediate vicinity of the vibrator, which is absent in Rudenberg's method.

We have shown above that a similar consideration of the problem may be easily realized with the help of the method of induced e. m. f.' s. According to this method, an oscillating vibrator is equivalent to a closed oscillatory circuit with its parameters determined by formulae (1, 14), (3), (6) and a foreign e. m. f. applied from outside $\mathcal{E}_f = -\mathcal{E}_z$. As regards the possibi-

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* Discussing the deduction of this formula, R. Becker [21] writes as follows: «Our deduction of the formula of «radiant brake-effect» remains highly unsatisfactory inasmuch as it fails to show in what manner the radiated spherical wave comes to influence electron motion... It was only by means of introducing a number of special admissions that Lorentz was able to obtain an approximate solution which, as one could easily expect, coincided with the solution obtained by us simply on the basis of the law of conservation of energy». [See H. Lorentz [22]].

lity of obtaining the same result with the help of Pointings' vector method, we propose to revert to this problem later on, since it entails that «reactance» energy fluxes must be taken into account.

Advice and criticism received from M. W. Shoulejkin and Dr. E. G. Kliatzkin in the course of this work are greatfully acknowledged by the author.

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Received
23. IV. 1939.

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Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1939. Volume XXIII, № 9

TECHNICAL PHYSICS

ELECTROMAGNETIC FIELD OF PARABOLOID-OF-REVOLUTION REFLECTORS

[By B. A. WWEDENSKY, Corresponding Member of the Academy, and E. N. MAIZELS

1. The directivity of microwaves is often obtained by an arrangement consisting of a metal sheet paraboloid of revolution with a radiator placed in the immediate vicinity of the focus of this paraboloid («mirror»). Accurate calculation of the field intensity thus obtained on the basis of electromagnetic theory is a matter of great difficulty. If, however, the size of the mirror is great enough (about 5 to 10 wave lengths λ), the „optical" theory becomes practically applicable; then the radiated field is calculated as a diffraction field by usual optical methods. This method has been adopted by Darbord (1), Morita (2) and Staal (3), but their considerations are open to some objections.

2. Take the axis of the paraboloid of revolution for the z axis and let the radiator be an elementary doublet, placed precisely at the focus. Then, provided the mirror is large enough as compared to λ, the radiated wave after being reflected by the mirror may be considered as a plane wave „incident" on the aperture of the mirror. Thus, the problem is reduced to Frauenhoffer's diffraction problem. It must be taken into account that the amplitudes are different at different points of the wave front, depending upon the angle ϑ between the vibrator axis and the ray emitted by the vibrator.

Let us consider at first the case when the radiator axis is perpendicular to the mirror axis z and coincides with the x axis. The radiation field has then two components, E_x and E_y. If, however, the diffraction angles θ are not too large, the sole E_x component may be used; this will be quite true if we deal with a direction coinciding with the mirror axis ($\theta=0$). Then we have only to find out the distribution of E_x on the plane of the mirror opening and to calculate the diffraction field on the basis of this distribution. If the distance between the focus and any point on the surface of the mirror becomes $\gg \frac{\lambda}{2\pi}$, then the field intensity E may be determined by the expression:

$$E_\vartheta = \frac{60\pi J l}{\lambda r} \sin\vartheta\, e^{jkr}, \qquad (1)$$

where J is the m. s. r. current, l and r are the vibrator length and the distance from the radiator respectively.

From (1) we obtain the distribution of E_x amplitudes in the plane of the opening by taking into account the rotation of the vector E_ϑ, due to reflexion:

$$E_x = \frac{60\pi Jl}{\lambda} \frac{p^2 - \rho^2 \cos 2\varphi}{p^2 + \rho^2} \frac{2p}{\rho_0^2 + p^2}. \tag{2}$$

Here ρ and φ are polar co-ordinates on the plane of the opening (φ being measured from the x axis); p is the parameter of the paraboloid and ρ_0 — the radius of the mirror opening. Assuming the mirror to be made of a perfectly conducting material, we may neglect the decrease of amplitudes caused by reflexion. Moreover, we shall consider the plane of the mirror opening to be normal to the mirror axis.

3. Write the well known Frauenhoffer's diffraction formula for small θ angles in the following form:

$$E_\theta = -\frac{j}{\lambda} \cos\theta \frac{e^{jk(R+r)}}{R} \int_0^{2\pi} d\varphi \int_0^{\rho_0} \cdot E_x \rho d\rho e^{-jk\rho \sin\theta \cos(\varphi - \delta)}. \tag{3}$$

Here r and R are the distances from the vibrator to the plane of opening and from latter to the observation point respectively; $R \gg \rho_0$; θ and δ are the zenith and the azimuth angles of the observation point («diffraction angles»). The «direct radiation» may be neglected here.

After E_x is substituted from (2) into (3) and the factor $-je^{jk(R+r)}$ is eliminated, we obtain for a field at the observation point:

$$E_\theta = \frac{60\pi Jl}{\lambda^2 R} \frac{4\pi p^3}{\rho_0^2 + p^2} \cos\theta \{\Phi_1 + \Phi_2 \cos 2\delta\}, \tag{4}$$

where

$$\Phi_1 = \frac{1}{2\pi} \int_0^{\rho_0} \frac{\rho d\rho}{p^2 + \rho^2} \int_0^{2\pi} d\varphi \, e^{-jk\rho \sin\theta \cos(\varphi - \delta)}, \tag{5}$$

$$\Phi_2 = -\frac{1}{2\pi p^2 \cos 2\delta} \int_0^{\rho_0} \frac{\rho^2 d\rho}{p^2 + \rho^2} \int_0^{2\pi} \cos 2\varphi d\varphi \, e^{-jk\rho \sin\theta \cos(\varphi - \delta)}. \tag{6}$$

The integration by parts leads to a series of Bessel's functions with increasing order of magnitude. Values of Φ_1 and Φ_2 are given by curves in Fig. 1 for a case when the focus of the mirror is placed in the plane of the opening.

Let the field gain of the mirror η be defined by the ratio of the mirror field intensity to that, produced by a vibrator without mirror. In both cases the field is measured in the direction of the maximum radiation (mirror axis) and the currents are maintained constant. We have then from (4) as $\theta = 0$:

$$\eta = \frac{2\pi p^3}{\lambda (p^2 + \rho_0^2)} \ln \frac{p^2 + \rho_0^2}{p^2}. \tag{7}$$

The quantity η has a maximum, when $p \cong 1.4\rho_0$:

$$\eta_{max} = 0.82 \frac{\pi \rho_0}{\lambda}. \tag{8}$$

If the opening of the mirror has a fixed radius, the value of η goes down not only with the decrease, but also with the increase of the focal

distance; if we place the focus in the plane of the opening, we have

$$\eta = 0.69 \frac{\pi \rho_0}{\lambda}. \qquad (9)$$

As the focal distance (for $\rho_0 = \text{const}$) grows, the uniformity of the field distribution in the opening of the mirror is increased, whereas the field strength in the opening is diminished, since the optical path vibrator—reflecting surface—plane of opening becomes longer. The optimum having been passed at $\rho_0 \cong 1.4\rho_0$ the second (diminishing) effect begins to prevail. When $p < \rho_0$ Darbord's «zones nuisibles» occur.

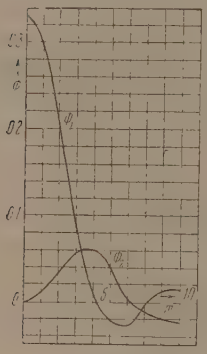


Fig. 1.

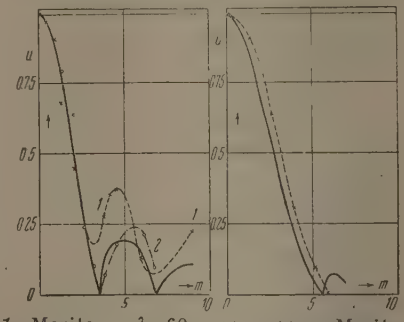
Fig. 2 shows the directional characteristics of a reflector with the focus in the plane of the opening. Reduced co-ordinates are used: $m = k\rho_0 \sin\theta$ and $u = \frac{E_\theta}{E_0}$, where E_0 corresponds to $\theta = 0$. The points are plotted according to Morita's and Staal's experiments, the conditions of their tests differing but little from those postulated by us.

4. To Darbord belongs the credit for stating and partly solving the problem considered here. He has already established the conception of „nuisance zones". When $\rho^2 \cos 2\varphi > p^2$ [eq.(2)], the sign before E_x is changed and hence E decreases owing to rays reflected from corresponding points of the reflector. It is obvious that no „nuisance zones" may occur when $\rho_0 < p$; that is the reason why Darbord has proposed to limit the mirror by a plane passing through the focus. It may be seen, however, from (7) that for an optimum case the boundary has to pass still nearer to the mirror apex.

The expression for E_x used by Darbord is

$$E_x = k2p \frac{p^2 - \rho^2 \cos 2\varphi}{(p^2 + \rho^2)^2}. \qquad (10)$$

It is easy to see that (10) refers to the mirror surface and not to the opening plane. Darbord ignores the fact that every point on the mirror surface has a different phase, though the optical paths between the vibrator (focus) and various points of the mirror surface are of different length. On the other hand, it cannot possibly be claimed that Darbord in his calculations proceeds from the plane of the mirror opening, for if it were so the denominator in (10) should then be $(p^2 + \rho^2)$ $2p$. In the case considered by Darbord the neglecting of the phase difference leads to an overrate of actual field gain by 3.2 db.



1—Morita, $\lambda = 68$ cm, $\rho_0 = 125$ cm, 2—Staal, λ-25 cm, $\rho_0 =$ 150 cm. Equatorial plane; $\times$ — $\times$ — Morita, $\lambda = 68$ cm, $\rho_0 = 125$ cm. Meridian plane

Fig. 2.

No calculations of radiation characteristics are available in Durbord's work, and Morita has set himself the purpose to fill in this gap. Yet he is using expression (10) without correcting it and therefore his curves lack in accuracy. Therefore the direction characteristics of Morita are much broader than those calculated according to (5) and (6) and even fail to agree with his own experimental results.

Staal's calculations are based on an obviously wrong statement that the parallel to the vibrator component of electric field is not affected by reflexion from the mirror. Indeed, his expression for this component in the plane of the mirror opening

$$E_{px} = 2pA \frac{4p^2y^2 + (p^2 - x^2 - y^2)^2}{(x^2 + y^2 + p^2)^3} \qquad (11)$$

is nothing else but the expression for a component of electric field, calculated for a case when the mirror is supposed to be absent; the corresponding optical path only is taken into account. Moreover, Staal's calculations of characteristics are based on a method of approximation which might readily be dispensed with. As was pointed out, a general solution as well as corresponding graphs may be obtained in a quite simple way.

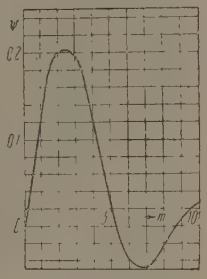


Fig. 3.

5. If the vibrator is placed along the reflector axis, the radiation characteristic is a figure of revolution. Fig. 3 refers to a case when a doublet is placed in the plane of opening. We have a maximum of radiation when the angle of radiation is

$$\theta_1 = \arcsin \frac{2.3\lambda}{2\pi\rho_0} \cong \frac{2.3\lambda}{2\pi\rho_0}. \qquad (12)$$

Then the field gain is nearly twice as low as that in (8), since

$$\eta_1 = 0.420 \frac{\pi\rho_0}{\lambda}. \qquad (13)$$

There are no «nuisance zones» in this case; the optimal value of η_1 is however at $p = \rho_0$.

6. In the case of a mirror with a hypothetical vibrator radiating uniformly in all directions:

$$E_\theta = \frac{60\pi Jl}{\lambda R} \frac{2p\rho_0}{\rho_0^2 + p^2} \frac{J_1(k\rho_0 \sin\theta)}{\sin\theta}, \qquad (14)$$

$$\eta = \frac{\pi\rho_0^2}{\lambda} \frac{2p}{\rho_0^2 + p^2}; \quad \eta_{\max} = \frac{\pi\rho_0}{\lambda}. \qquad (15)$$

7. We are giving here the following expressions for the main beams of directional characteristics (θ_0 being the angle between the direction of maximum radiation and that of the first «zero» radiation); a) a uniformly radiating vibrator $\sin\theta_0 = 1{,}22 \frac{\lambda}{2\rho_0}$; b) a normal vibrator (doublet) with its axis lying in the plane of the mirror opening for equatorial plane of the vibrator $\sin\theta_0 = 1.05 \frac{\lambda}{2\rho_0}$; for meridional plane of the vibrator

$$\sin\theta_0 = 1.73 \frac{\lambda}{2\rho_0} \qquad (*).$$

except, this case the values of θ_0 differ but little from corresponding values for an array antenna with equiphased vibrators whose total length is $2\rho_0$.

Received
28. IV. 1939.

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Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1939. Volume XXIII, № 9

PHYSIKALISCHE CHEMIE

ÜBER DIE ANWENDUNG KÜNSTLICHER RADIOELEMENTE ZUR AUFKLÄRUNG EINIGER FRAGEN DER KOMPLEXCHEMIE

Von A. A. GRÜNBERG und P. M. FILINOW

(Vorgelegt von W. G. Chlopin, Mitglied der Akademie, d. 9. III. 1939)

Wir haben versucht, künstliche Radioelemente und vorerst das Radiobrom zur Klärung der Fragen über den Zustand der Komplexionen in der Lösung, sowie über die Gleichwertigkeit der durch Haupt- resp. Nebenvalenzkräfte festgehaltenen koordinierten Liganden, anzuwenden.

Die erste Frage besteht nämlich darin, ob die Komplexionen vom Typus $[PtBr_4]^=$ oder $[PtBr_6]^=$ in der Lösung des Elektrolyten KBr unverändert bleiben, oder ob sie das koordinierte Brom gegen die in der Lösung befindlichen Bromionen stetig austauschen.

Die zweite Frage schien schon längst seitens der Komplexchemiker im Sinne der Gleichwertigkeit aller gleichartiger koordinierter Liganden gelöst zu sein. Die neueren Angaben von S. Z. Roginsky (1) über die Ungleichwertigkeit der beiden Bromatome in $CuBr_2$, sowie einige Angaben von Drew und seinen Mitarbeitern, liessen diese Frage nochmals auftauchen.

Indem wir als Indikator das durch die Neutron-Bestrahlung von $C_2H_4Br_2$ gewonnene und durch Ausschüttelung mit einer verdünnten KBr-Lösung in wässrige Lösung überführte Radiobrom benützten, konnten wir die beiden aufgeworfenen Fragen eindeutig beantworten.

Es ist nämlich gelungen zu zeigen, dass sowohl K_2PtBr_4 wie auch K_2PtBr_6 in einer KBr enthaltenden Lösung die koordinierten Bromionen gegen die in der Lösung vorhandenen freien Br-Ionen stetig austauschen. Der Austauschvorgang erfolgt mit grosser Geschwindigkeit und führt zur Einstellung des Gleichgewichtszustandes. Enthält eine wässrige Lösung $0.4 \cdot 10^{-2}$ Mol $K_2[PtBr_4]$, $0.7 \cdot 10^{-2}$ Mol KBr und ca. 10^{-17} Mol Radiobromid, so wird der Gleichgewichtszustand weniger als während 1 Stunde 45 Min. (bei 16°C) erreicht.

Die zur Einstellung des Gleichgewichts notwendige minimale Zeitdauer wurde bisher noch nicht genau festgestellt. Wir konnten aber zeigen dass schon nach 20 Min. der Austausch mit dem Radiobromid ungefähr 80% des dem Gleichgewichtszustande entsprechenden beträgt. Mit K_2PtBr_6 wurden die Austausch-Versuche beim Sieden ausgeführt, da in der Kälte die Löslichkeit des Komplexes gering ist.

Die Versuche mit K_2PtBr_4 wurden in folgender Weise angestellt: das bestrahlte Äthylenbromid wurde mit einer verdünnten wässrigen KBr-Lösung (16 mg KBr pro 20 cm^3) während ca. 5 Min. stark geschüttelt. Die Aktivität der so erhaltenen Lösung wurde entweder durch Behandlung

von 20 cm³ mit überschüssigem $AgNO_3$ und darauffolgende Ausmessung der AgBr-Aktivität, oder durch Verdunsten von 20 cm³ der Lösung und Messung der Aktivität des trockenen Rückstandes bestimmt. Die Messung wurde mit einem Zähler vom Typus Geiger-Müller ausgeführt.

Die 20 cm³ Portionen der «titrierten Radiolösung» wurden mit bestimmten Einwaagen von K_2PtBr_4 versetzt und einige Zeitlang bei 16°C stehen gelassen. Darauf wurde der Lösung $[Pt(NH_3)_4]$ SO_4 hinzugefügt, wodurch das Ion $[PtBr_4]^-$ in Form von $[Pt(NH_3)_4]$ $[PtBr_4]$ niedergeschlagen wurde (die Ausbeute des Niederschlags beträgt etwa 95% der theoretisch berechneten). Das Filtrat nach der Abscheidung des Salzes $[Pt(NH_3)_4]$ $[PtBr_4]$ wurde entweder mit $AgNO_3$ bearbeitet (Abscheidung von AgBr) oder ganz einfach ins Trockene eingedampft. Durch die Messung der Aktivität von $[Pt(NH_3)_4]$ $[PtBr_4]$ einerseits und des aus dem Filtrat gewonnenen Rückstandes andererseits liess sich die Verteilung des Radiobroms zwischen den Bromionen innerhalb und ausserhalb des Komplexes direkt finden.

Es seien einige Zahlenangaben angeführt. Die Aktivitäten sind in Ausschlägen des Zählers, wobei ein jeder Ausschlag acht Entladungen entspricht, ausgedrückt. Die Aktivität von 20 cm³ der «Radiolösung», die 16 mg KBr (11 mg Br^-) und etwa 10^{-17} Mol des Radiobromids enthielt, betrug 354.

a) Wurden in 20 cm³ dieser Lösung 0.05 g K_2 $PtBr_4$ (27 mg Br^-) aufgelöst (die Zeitdauer des Versuchs war 1 St. 45 Min. bei 16°C), so betrug die Aktivität von $[Pt(NH_3)_4]$ $[PtBr_4]$—254 und die Aktivität von AgBr aus dem Filtrat—132. Insgesamt betrug die Aktivität beider Portionen—386.

b) Beim Auflösen in 20 cm³ derselben Lösung von 0.05 g K_2PtBr_4 und noch 64 mg KBr (ceteris paribus), wurde die Aktivität des Komplexes gleich 125, die Aktivität der Mutter-Lösung gleich 197 gefunden. Die Summa der Aktivitäten war diesmal gleich 322. Der Gehalt der Lösung an KBr war hier gleich 80 mg (55 mg Br^-).

Die angeführten Data geben uns die Möglichkeit ausser der Feststellung des Austausches noch die Zahl der im Komplexion enthaltender Bromionen, die am Austausche beteiligt sind, zu bestimmen.

Werden alle vier Bromionen des Komplexes ausgetauscht, so muss die Aktivität des Komplexes $\frac{27}{38}$ oder 70% der totalen Aktivität betragen (Versuch a). (Das Radiobrom wird auf alle im System vorhandenen und an dem Austausch beteiligten Bromatome gleichmässig verteilt). Im Falle des Versuchs b) soll die in derselben Weise berechnete Aktivität des Komplexes $\frac{27}{82}$ oder 33% der totalen Aktivität betragen. Es wurde gefunden: 66% (Versuch a) und 39% (Versuch b).

Werden nur 2 Bromionen des Komplexes ausgeutauscht, so müssten wir haben 55% (Versuch a) und 19% (Versuch b). Somit ist es klar, dass alle vier Bromionen von $[PtBr_4]^=$ austauschbar sind. Es war schon erwähnt dass beim Konzentrationsverhältniss, das für den Versuch a) charakteristisch ist, während 20 Min. etwa 80% der Gleichgewichtsmenge des Radiobroms in den Komplex eintritt.

Da der Komplex im Gleichgewichtszustande etwa 70% des in der Lösung vorhandenen Radiobroms enthält, so liegt es nahe, dass die Menge des vom Komplex aufgefangenen Radiobroms schon nach Verlauf von 20 Min. ($1.2.10^3$ Sek.) mit der Grössenordnung von 10^{-17} Mol (per Liter) zu schätzen ist. Die Menge des inaktiven Broms in der Lösung ist ungefähr 10^{15} mal grösser, und so ergibt es sich, dass der Austausch von 10^{-17} Mol inaktiven Bromids mit dem Komplex schon in $\frac{1.2 \cdot 10^3}{10^{15}} = 10^{-12}$ Sek. erreicht wird.

Mit anderen Worten, tauschen sich mit dem Komplex während einer Sekunde unter dem gegebenen Verhältniss der Konzentrationen 10^{-5} Mol

des inaktiven Bromids aus, d. h. etwa 0.1% der in der Lösung vorhandenen Menge.

Im Falle von $K_2[PtBr_6]$ lässt sich die Frage nach der Zahl der am Austausch beteiligten Bromionen des Komplexes durch die Zusammenstellung der Versuchsdata mit den theoretisch errechneten Koeffizienten der Aktivitätsverteilung nicht direkt entscheiden. Die experimentell ermittelte Aktivitätsverteilung zwischen dem Komplex und der Mutter-Lösung steht sowohl mit der Annahme der Austauschbarkeit von 6 Bromionen, wie auch mit derjenigen für 4 Bromionen, in Übereinstimmung. Die unter Zugrundelegung beider Annahmen errechneten Koeffizienten sind voneinander sehr wenig verschieden.

Um diese Frage doch endgültig entscheiden zu können, verfuhren wir in folgender Weise: es wurde ein aktiviertes $K_2[PtBr_6]$ Präparat durch Umkristallisieren aus der «Radiolösung» eines inaktiven Präparats dargestellt. Darauf wurden zwei identische Einwaagen dieses aktivierten Präparats vorbereitet (zu 0.05 g). Die Aktivität einer Probe wurde direkt gemessen: sie betrug 68. Die andere Probe wurde einer quantitativen Reduktion zu K_2PtBr_4 unterworfen. Das erhaltene $PtBr_4$ Ion wurde auch diesmal in Form von $[Pt(NH_3)_4][PtBr_4]$ abgeschieden. Die Mutter-Lösung nach der Abscheidung von $[Pt(NH_3)_4][PtBr_4]$ wurde ins Trockene eingedampft. Die Aktivität des $[PtNH_3][PtBr_4]$ war gleich 43, die Aktivität des aus der Mutter-Lösung gewonnenen Rückstandes betrug 19 (totale Aktivität = 62).

$$\frac{\text{Aktivität von } [PtBr_4]^-}{\text{Aktivität von } [PtBr_6]^-} = \frac{43}{68} = 0.63.$$

Im Falle der Gleichwertigkeit aller sechs Bromionen in $[PtBr_6]^-$, soll die Aktivität von $[PtBr_4]^-$ zwei Drittel der Aktivität des Ausgangskomplexes, oder 0.67, betragen.

Somit liess sich die Austauschbarkeit aller Bromatome in $[PtBr_6]^-$ direkt beweisen. Daneben kann auch die Gleichwertigkeit aller drei Koordinaten im Oktaeder von $PtBr_6$ als höchst wahrscheinlich gelten.

Die erhaltenen Resultate erlauben uns der Lösung einer ganzen Reihe der Problemen der Komplexchemie näher zu treten. Darüber wird in einer ausführlichen Arbeit später berichtet. Wir halten es für eine angenehme Pflicht dem Vorstand des Radiuminstituts, Herrn Akademiker W. G. Chlopin, sowie M. A. Passwik-Chlopina, A. E. Polessitsky und N. F. Wolkow auch an dieser Stelle für das freundliche Entgegenkommen beim Ausführen dieser Arbeit unsern besten Dank auszusprechen.

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des Chemisch-Technologischen Instituts.
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Wissenschaften d. UdSSR.
Leningrad.

Eingegangen
d. 13. III. 1939.

ZITIERTE LITERATUR

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Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1939. Volume XXIII. № 9

CHEMIE

DER ÜBERGANG VON KOHLENHYDRATEN ZU KARBOZYKLISCHEN VERBINDUNGEN. I. DIE UMWANDLUNG DER GLUKOSE IN PHENOL*

Von P. SCHORIGIN, Mitglied der Akademie, und N. N. MAKAROWA-SEMLJANSKAJA

In der lebendigen Natur, in der Pflanzenwelt, finden ständige Umwandlungen von Kohlenhydraten, dieser primären Assimilationsprodukten der Luft-Kohlensäure, in mannigfaltigste zyklische Verbindungen statt: wir finden nämlich in verschiedenen Pflanzenteilen Harze, Terpene, Gerbstoffe, Farbstoffe, Sterine u. s. w. Zur aromatischen Reihe gehört auch die typische inkrustierende Substanz der pflanzlichen Zellwänden—Lignin, welches ca. $^1/_3$ vom Gewicht des trockenen Holzes ausmacht. Der genetische Zusammenhang von allen diesen zyklischen Substanzen mit Kohlenhydraten ist ausser Zweifel—sie bilden sich im Pflanzenorganismus infolge biochemischer Umwandlungen von Kohlenhydraten, die dabei wahrscheinlich verschiedene Zwischenstufen verlaufen.

Die meisten von obenerwähnten natürlichen zyklischen Verbindungen enthalten isolierte oder kondensierte Ringe aus 6 Kohlenstoffatomen, einige von ihnen—sechsgliederige Ringe, die mit fünfgliederigen karbozyklischen (z. B. Sterine), oder mit sechsgliederigen sauerstoffhaltigen heterozyklishen (Pflanzenfarbstoffe der Flavon- und Flavonolgruppe, Anthozyane, Katechine u. s. w.) kondensiert sind. Die Kohlenhydrate selbst gehören zu heterozyklischen Verbindungen (Pyranosen, seltener Furanosen).

Im Erdöl, diesem geologischen Umwandlungsprodukt von pflanzlichen Kohlenhydraten, befinden sich wieder fünf-, bzw. sechsgliederige zyklische Kohlenwasserstoffe (Naphtene).

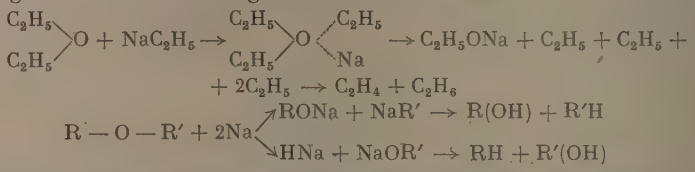
Die Anwesenheit in natürlichen Kohlenhydraten von Ketten mit 5 und 6 Kohlenstoffatomen beweist, dass die Zyklisierung solcher Ketten nicht nur in lebendiger Natur, sondern auch bei geologischen Umwandlungen von Pflanzenresten stattfindet. Bisher war es aber noch niemals den Chemikern gelungen, die Kluft zwischen Kohlenhydraten und aromatischen Verbindungen zu überbrücken, abgesehen von der Bildung sehr kleiner Mengen der letzteren bei trockener Destillation der Zellulose, die ca. 4% vom Teer gibt, der geringe Menge phenolischer Substanzen enthält. Marr muss dabei beachten, dass pyrogenetische Zersetzung nur bei hoher Temperatur (250—400°) und in sehr komplizierter Weise vor sich geht, dass ihr Mechanismus vollkommen unübersichtlich ist, und dass dieser Prozess allerdings nicht als Übergang von einer Körperklasse in die andere betrachtet werden kann.

Die Erfindung von klaren, glatt verlaufenden Reaktionen, die eine Umwandlung von Kohlenhydraten in aromatische, oder überhaupt in karbozy-

* Mitgeteilt in der Sitzung der Chemieabteilung der Akademie am 27 März 1939.

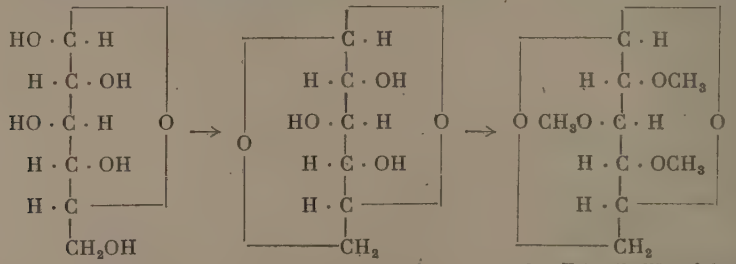
klische Verbindungen erlauben, wäre von grösstem Interesse nicht nur in rein wissenschaftlicher Beziehung, vom Standpunkte der besseren Erforschung von Prozessen, die in den Pflanzen und in der Erdkruste bei der Mineralisierung von Pflanzenresten vor sich gehen, sondern auch im Sinne möglicher Ausnutzung dieser Reaktionen zur praktischen Darstellung von aromatischen Verbindungen, die bekanntlich so ausgedehnte Anwendung in der derzeitigen chemischen Industrie finden. Es wäre z. B. sehr verlockend Phenol aus Kartoffel, oder Benzel und Toluol aus Holzabfällen zu fabrizieren. Bei sehr hoher Entwicklung, welche die organische Chemie heute erhalten hat, wenn man z. B. die azyklischen Kohlenwasserstoffe des Erdöls nicht nur in Laboratorien, sondern auch in technischem Masstabe aromatisiert, zersplittert, verzweigt u. s. w., kann man hoffen, dass schon in nicht so entfernter Zukunft die neue technische Ausnutzung von natürlichen Kohlenhydraten—ihre Umwandlung in wertvolle aromatische Verbindungen—verwirklicht wird.

Unsere Arbeit stellt einen ersten erfolgreichen Versuch in dieser Richtung vor. Wir benutzten dabei die bereits vor vielen Jahren von Schorigin [1] erfundenen Reaktionen—die Zersetzung von Äthern durch die Einwirkung von Na-organischen Verbindungen und metallischem Na:



Über neuere Arbeiten in dieser Richtung, unter Anwendung von Na-Lösungen in flüssigen NH_3 war von P. Schorigin und Makarowa-Semljanskaja [2] und von P. Schorigin und Skoblinskaja [3] in der Sitzung von OMEN der Akademie im Januar 1937 berichtet. Während folgender 2 Jahre haben wir unsere Untersuchung weiter fortgesetzt und wir beschreiben in vorliegender Mitteilung nur einen beschränkten Teil von ausgeführten Arbeiten, nämlich die Karbozyklisierung von Kohlenhydraten.

Als Ausgangsstoff haben wir Trimethyllävoglükosan angewandt. Pictet [4] hatte bekanntlich gezeigt, dass Lävoglukosan bei der Vakuumdestillation von Stärke, Cellulose und β-d-Glukose das Hauptprodukt ist. Karrer und Smirnoff [5] ist es gelungen Lävoglukosan auch synthetisch (aus Acetobromglukose) darzustellen. Trimethyllävoglukosan wurde von Irvine und Oldham[6] durch Methylierung von Lävoglukosan mit Hilfe von Dimethylsulfat in Gegenwart von NaOH erhalten. Als Ausgangsstoffe für diese Verbindung können also Zellulose, Stärke und Glukose dienen.



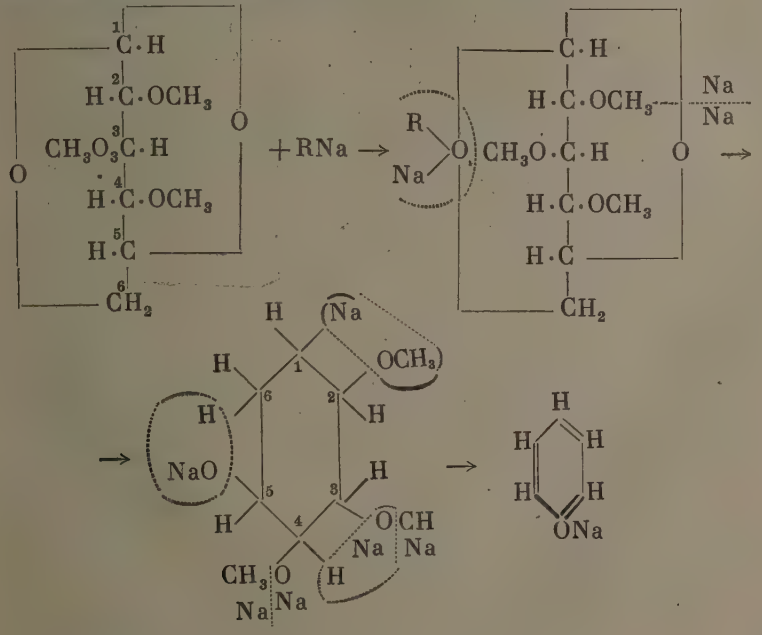
β-d-Glukose — Lävoglukosan — 2, 3, 4-Trimethyllävoglukosan

Bei der Einwirkung von Na-Lösungen in flüssigen NH_3 auf Trimethyllävoglukosan bei Zimmertemperatur haben wir mit verhältnismässig guter Ausbeute (ca. 24% der theoretischen) reines Phenol $C_6H_5(OH)$ erhalten.

Möglicherweise wurden dabei zweiwertige Phenole gebildet, denn der Nachlauf des Reaktionsproduktes zeigte intensive grüne Färbung mit $FeCl_3$

Wir bringen hier die Beschreibung eines von diesbezüglichen Versuchen, die immer dieselben Resultate gaben; 3.86 g krystallinischen Trimethyllävoglukosan (Schmp. 65°) wurden in 45 cm^3 von trockenem flüssigem NH_3 (in einem Zuschmelzrohr, das in einem Dewarschen Gefäss mit fester CO_2 sich befand) aufgelöst und zu erhaltener Lösung 2.7 g Na (theoretische Menge, berechnet für die Abspaltung von 3 Methoxylgruppen, beträgt 2.61 g Na) almählich zugesetzt. Am folgenden Tage wurde das Rohr zugeschmolzen und bei Zimmertemperatur stehen gelassen. Während der nächsten Tage nahm der Rohrinhalt intensive rote Farbe, die durch die Bildung von Na-organischen Verbindungen verursacht war, an; die Farbe verschwand aber bei weiterem Stehen und das Reaktionsprodukt verblieb dann farblos. Die blaue Lösung entfärbte sich nach ca. 18 Tagen und im Rohr befand sich dann ein gelblicher Bodensatz. Das Rohr wurde wieder abgekühlt und geöffnet, NH_3 abgedampft, das Reaktionsgemisch zuerst mit wasserhaltigem Äther, dann mit Wasser bearbeitet; die alkalische wässrige Lösung wurde angesäuert und ausgeäthert, ätherischer Auszug wurde mit Na_2SO_4 getrocknet: nach dem Abtreiben des Äthers erhielt man als Rückstand 1.56 g zähflüssige, nach Phenol intensiv riechende Flüssigkeit. Beim Destillieren unter gewöhnlichem Druck wurden 3 Fraktionen erhalten: I) bis 180°—0.2 g; II) 180—183°—0.55 g; III) 183—210°—0.2 g; im Kolben verblieb eine kleine Menge harziger Substanz. Alle drei Fraktionen waren viskose gelbliche Flüssigkeiten mit intensivem Phenolgeruch; die zweite Fraktion—reines Phenol—erstarrte krystallinisch beim Abkühlen und gab beim Behandeln mit C_6H_5COCl in NaOH-Lösung das für einfaches Phenol charakteristische krystallinische Benzoat, Schmp. 68—69°. Die wirkliche Menge von gebildetem Phenol muss bedeutend grösser sein, denn infolge kleiner Menge des angewandten Trimethyllävoglukosan sollen relative Verluste verhältnismässig gross sein.

Nach unserer Voraussetzung kann man den Mechanismus dieser Reaktion etwa folgenderweise formulieren:



Der Ringschluss zwischen C_1 und C_6 erfolgt durch Anlagerung von Na-organischen Verbindungen, die im Laufe der Reaktion gebildet werden (z. B. CH_3Na), an die Sauerstoffbrücke (1.6) mit darauffolgender Fortnahme von O-Atom. Das Entstehen von Doppelbindungen zwischen C_1 und C_2, sowie zwischen C_3 und C_4, kann man auf Grund obenerwähnter Reaktionen von P. Schorigin erklären. Die Möglichkeit des Entstehens der Doppelbindung zwischen C_5 und C_6 durch Abspaltung von NaOH folgt aus den (vorläufig unveröffentlichten) Versuchen von P. Schorigin und N. W. Schorigin, die Styrol mit guter Ausbeute aus β-Phenyläthylalkohol folgenderweise erhielten:

$$C_6H_5 \cdot CH_2 \cdot CH_2OH + Na \rightarrow C_6H_5 \cdot CH_2 \cdot CH_2ONa + H$$
$$C_6H_5 \cdot CH_2CH_2 \cdot ONa \rightarrow C_6H_5 \cdot CH = CH_2 + NaOH$$
$$C_6H_5 \cdot CH_2 \cdot CH_2(OH) + NaOH \rightleftarrows C_6H_5 \cdot CH_2 \cdot CH_2ONa + H_2O$$

Es erwies sich nämlich, dass beim Erhitzen von 1 Mol β-Phenyläthylalkohol mit 0.5 g-Atom Na Styrol mit Ausbeute von ca. 75% der Theorie, bei Anwendung von 0.1 g-Atom Na—mit Ausbeute von ca. 60% entsteht.

Selbstverständlich darf man diese von uns aufgefundene Reaktion keinesfalls als eine praktisch brauchbare Methode der Umwandlung von Kohlenhydraten in aromatische Verbindungen betrachten, aber ihre wissenschaftliche Bedeutung unterliegt keinem Zweifel—man kann sie als ersten gelungenen Übergang von Zuckerarten zu karbozyklischen Stoffen betrachten; die Reaktion ist desto bemerkenswerter, dass sie schon bei gewöhnlicher Temperatur vor sich geht. Es lohnt sich kaum zu erwähnen, dass in den Pflanzen die Umwandlungen von Kohlenhydraten nach anderen Wegen und unter Mitwirkung von Enzymen geschehen, in Erdkrusten aber bei der Einwirkung von möglicherweise sehr schwachen, aber während Millionen von Jahren wirkenden chemischen Agentien, vor sich gehen.

Man muss auch bemerken, dass die von uns erhaltenen Resultate möglicherweise zugunsten bekannter Anschauungen von Hilpert ausgelegt werden könnten. Der letztere verneint überhaupt die Existenz vom genuinem Lignin in verholzten Zellwänden und betrachtet diese Substanz als ein Umwandlungsprodukt von Kohlenhydraten, bedingt durch die für seine Isolierung angewandten Bearbeitungen. Wir sind aber von anderer Meinung und hatten uns schon früher gegen diese Behauptung von Hilpert ausgesprochen [1]. Die vorliegende Arbeit bestätigt nur den genetischen Zusammenhang zwischen Kohlenhydraten und Lignin in Pflanzenzellwänden, keinesfalls wird aber dabei die Existenz des genuinen Lignins in Frage gestellt.

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Eingegangen
d. 25. IV. 1939.

ZITIERTE LITERATUR

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Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1939. Volume XXIII, № 9

CHEMISTRY

STUDY OF THE ISOMERIZATION OF BUTANES AND THEIR EQUILIBRIUM RATIOS

By B. L. MOLDAVSKIJ and T. V. NISOVKINA

(Communicated by A. N. Terenin, Member of the Academy, 20.IV.1939)

In the present paper are reported the results of experiments on the catalytic isomerization of butanes.

The isomerization of n-butane to isobutane and vice versa as due to the effect of aluminium chloride has been studied and the equilibrium composition of the mixture n-butane—isobutane determined in the temperature range of 70 to 180°.

There are very few experimental data available on the isomerization of butanes. An experiment is described ([1]) where isobutane was produced by means of heating n-butane in the autoclave at 175° in the presence of aluminium chloride, but the greater part of n-butane was decomposed. Montgomery, Mc Ateer and Frank in their short paper ([2]) describe the experimental determination of the n-butane $\leftrightarrows$ isobutane liquid-phase equilibrium at 27° with the $AlBr_3$ catalyst. According to the data of the above authors, the equilibrium mixture of butanes at this temperature contained in the liquid phase 78—82% of isobutanes and 18—22% of n-butane.

The theoretical computation of the equilibrium of butane isomerization reactions has been carried out by several authors. Kassel ([3]) has calculated the equilibrium constant from spectroscopic data assuming that a free rotation takes place about the C—C bond. The same calculations were afterwards repeated by Pitzer ([4]) taking into consideration the fact that the rotation about the C—C bond is not a free rotation but a hindered one.

Parks, Shomate, Kennedy and Crawford Jr. ([5]) have computed the entropies of n-butane and isobutane from the third law of thermodynamics and found a certain discrepancy with the spectroscopic calculations. Serebriakova and Frost ([6]) have calculated the equilibrium constant of butane isomerization indirectly, from the equilibrium constants of butylene isomerization as found experimentally and from the equilibrium constants for butane dehydrogenation as given by Frey and Huppke. They point at the great difference between the K_p values as calculated from their data and from those of Kassel.

The butanes used for the experiments were prepared from the corresponding alcohols by means of hydrogenating them over molybdenum sulfide.

Aluminium chloride was prepared from the commercial product by means of distilling it in a sealed tube in the presence of aluminium dust. The experiments on isomerization were carried out at 70° in liquid phase and at 110° to 180° in vapour phase. In the liquid phase experiments butanes were shaken with the catalyst $AlCl_3+CuSO_4 \cdot 2HCl$ in a sealed ampulla in a water thermostate, and in the vapour phase butane vapours together with hydrogen chloride were passed over $AlCl_3$, placed in a glass reaction vessel

heated on a glycerol bath. The experiments on the determination of the equilibrium were carried out from the butane side as well as from that of the isobutane.

In the experiments on hydrogenation at 70 and 110° no formation of by-products has been observed. In the experiments at higher temperatures the decomposition of butanes took place, along with isomerization, especially when working with isobutane, propane being formed as the principal product.

°C	% of iso-C_4H_{10} in the equilibrium mixture	$K_p = \frac{\%\text{ of iso-}C_4H_{10}}{\%\text{ of n-}C_4H_{10}}$	lg K_p
70	79	3.8	0.580
110	72	2.6	0.415
130	67	2.0	0.301
150	63	1.7	0.230
180	58	1.4	0.146

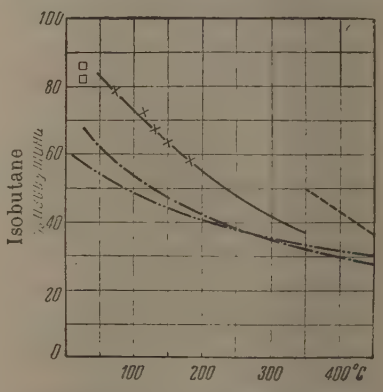
The analysis of the resulting mixtures of isomeric butanes has been carried out by means of fractionation in the apparatus of the Podbielniak type modified according to indications in the paper by Mc. Millan (7).

In the table the mean percent values of isobutane are given as contained in the equilibrium mixture with n-butane. As the experiments at 70° have been carried out in the liquid phase, the results are calculated for the vapour phase according to the Raoult's law.

In the same table, the calculated equilibrium constants are given for the vapour phase isomerization n-butane $\rightleftarrows$ isobutane in the temperature range of 70 to 180°. The dependence of K_p on temperature is expressed by the equation:

$$\lg k_p = \frac{611}{T} - 1.204. \qquad (1)$$

Fig. 1 shows the dependence on temperature of the equilibrium percentage of isobutane in the mixture with n-butane as computed from the equation (1). For comparison, the curves calculated from the data of Serebriakova and Frost, Kassel and Pitzer are also given as well as the equilibrium isobutane percentage as determined experimentally in the liquid phase at 27° by Montgomery, Mc Ateer and Frank and calculated by us for the vapour phase, according to the Raoult's law. As may be seen from the curves, experimental data of these authors are very near to our results. As regards the theoretical equilibrium calculations made by them considerable discrepancy between these and our experimental results is observed which indicates the presence of errors in theoretical calculations.



Equilibrium concentration of isobutane in a mixture with *n*-butane, as dependent on temperature

× × × The data of the authors
——— Calculated from equation I
Calculated from data:
— — — Serebriakova and Frost
— . — . Kassel
·····Pitzer
□ Montgomery and co-workers

State Chemical Institute
of High Pressure

Received
4. V. 1939.

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Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1939. Volume XXIII, № 9

CHEMISTRY

CONVERSION OF ASTRAKHANITE

By I. G. DRUZHININ

(Communicated by N. S. Kurnakov, Member of the Academy, 29. III. 1939)

Expeditionary and research work carried out for the last ten years by the Institute of General and Inorganic Chemistry of the Academy of Sciences allowed to establish the existence of a large number of salt lakes with primary deposits of the mineral astrakhanite $Na_2SO_4MgSO_4.4H_2O$, estimated at millions of tons. The want for sodium sulphate as the principal raw material for the glass and chemical industry may be partly satisfied by working over astrakhanite to Na_2SO_4 and magnesium salts by the method suggested by V. I. Nikolaev (private information).

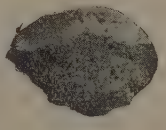

Fig. 1.

We undertook the present work in order both to give a more detailed characteristics of the properties of astrakhanite and to find out the conditions necessary to set going the process of astrakhanite treatment; in our investigation we had recourse to the thermal method. The heating curves of astrakhanite were been recorded after the method adopted in this Institute, i. e. using a Kurnakov's recording pyrometer and a differential thermocouple. The heat of the substance to be tested placed in a porcelain crucible was fixed by a simple thermocouple (Pt+PtRh). The difference in temperature arising in the exo- and endothermal processes connected with the chemical conversion of the substance under investigation was recorded by a differential thermocouple (Pt+PtRh) immersed into an indifferent substance (MgO).

The rate and regularity of heating of the substance under test in a cascade stove has been controlled by means of a potential regulator. The rate of heating applied was six degrees a minute.

The material investigated was of a good crystalline form (Fig. 1). The crystals of synthetic astrakhanite were transparent while those of the natural mineral had a dark colouring due to the presence of impurities: slime, sand, etc. By their chemical composition the preparations of natural minerals consisted of 97.98% $Na_2SO_4MgSO_4.4H_2O$. Heating curves have been obtained for specimens both of synthetic and natural astrakhanites from the lake Khodzhamsayat (Middle Asia), the lakes Bassinsk (Astrakhan region) and the Karabogaz gulf. The results of the experiments are summarized in the table.

Specimens	Conversion temperatures				Temperature of fusion	Notes
	Astrakhanite into löweite	Löweite into anhydrous astrakh.	Polymorph. convers. into anhydrous astrakhanite	Addition. effects connected with impurit.		
Synthetic astrakhanite	105	215	625	—	670	
Natural astrakhanite from the lake Khodzhamsayat . . .	105	215	590	575	625	Impurity — kieserite
Astrakhanite from the lake Bassinsk . . .	100	215	605	—	655	
Astrakhanite from the Karabogaz gulf . .	100	215	590	180	620	Impurity — gypsum

The heating curves of astrakhanite permit of the conclusion that within the temperature range from 25 to 700° this salt has four endothermal effects (Fig. 2). At 105° astrakhanite suffers a chemical conversion into löweite accompanied by the isolation of two water molecules according to the reaction:

$$Na_2SO_4 \cdot MgSO_4 \cdot 4H_2O \rightarrow Na_2SO_4 \cdot MgSO_4 \cdot 2H_2O + 2H_2O.$$

The process of conversion of astrakhanite into löweite is completed at 110°, above which a regular heating of löweite takes place. The second endothermal effect is observed at 215°, being connected with the loss of hydrated water by löweite and the passing of the latter to an anhydrous salt. The loss of water proceeds for 12—15 min. with a high heat absorption. A regular rise of the curve after 215° is to be noted, which should be expected to record the polymorphous conversion of sodium sulphate and the endothermal process of magnesium sulphate at as high a $t°$ as 575—580°. According to N. S. Kurnakov and S. Z. Makarov ([1]), sodium sulphate undergoes a polymorphous conversion at 240°. The absence of thermal effects over this part of the curve may evidently be accounted for by the formation of a chemical compound in equimolecular quantities of Na_2SO_4 and $MgSO_4$. The stop at 625° points to a polymorphous conversion of the chemical compound $Na_2SO_4.MgSO_4$, while at 670° it corresponds to the melting of this salt.

The formation of the chemical compound $Na_2SO_4.MgSO_4$ and its polymorphous conversion is confirmed by the cooling curves, the stops being somewhat behind. Crystallization proceeds at 665° and the polymorphous conversion at 620° (Fig. 2, part of the cooling curve).

Speranskaja ([2]) who studied the Na_2SO_4—$MgSO_4$ system by the fusibility method, has also observed the formation of the chemical compound $Na_2SO_4.MgSO_4$, to which she gives the name of gamma-modification of astrakhanite. She points out that anhydrous astrakhanite is formed below the temperature of eutectic (678°) undergoing two more polymorphous conversions at 276—280° and 480—500° respectively. Studying astrakhanite by the thermal method, the present author succeeded in establishing that the formation of the anhydrous salt $Na_2SO_4.MgSO_4$ goes on at 215° simultaneously with the process of löweite dehydration. I failed to observe the polymorphous conversions of anhydrous astrakhanite, found by Speranskaja at 280 and 500°. The temperature 480—500° of the polymorphous conversion should be referred rather to a partial process of disintegration of anhydrous astrakhanite and formation of vanthoffite.

The heating curves of specimens of natural astrakhanite from the lake Khodzhamsayat, the lakes Bassinsk and the Karabogaz gulf show all the endothermal stops characteristic of astrakhanite with a slight fall of temperatures due to the presence of impurities. The curve of astrakhanite from lake Khodzhamsayat has a small additional effect at 575° evidently corresponding to the free magnesium sulphate, 1% of which is present in the astrakhanite as an impurity. For the specimen from Karabogaz an additional stop is observed at 180°, corresponding to the dehydration of gypsum. Löweite and anhydrous astrakhanite have been prepared at 110—125° and 300—400°,

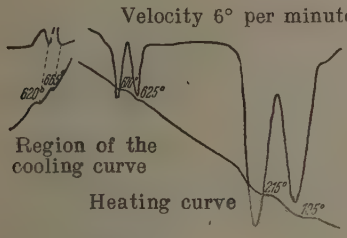

Fig. 2.

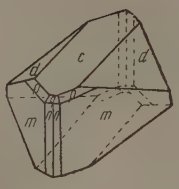

Fig. 3.

use being made of the experimental data obtained. In the course of determination of the chemical composition of löweite the following constituents in per cent by weight have been found:

	Mg,%	Na,%	SO_4,%	H_2O,%
Löweite obtained from synthetic astrakhanite	8.16	15.44	64.35	12.09
Löweite from natural mineral of lake Khodzhamsayat	7.98	14.71	62.10	11.50

In his classical works Vant-Hoff [3] points to that Tschermak, Hauer [4] and others, while determining the composition of löweite, have found in it up to 14.5% of combined water and accordingly ascribed to it the chemical formula $(Na_2SO_4.MgSO_4).5H_2O$.

The data obtained by us entitle us to maintain, contrary to Tschermak, Hauer, Haidinger [5] and others, that löweite contains only two molecules of water and has the chemical composition $Na_2SO_4.MgSO_4.2H_2O$.

To make the homogeneity of the prepared specimens of löweite and anhydrous astrakhanite more obvious as well as to confirm the assumption advanced, we carried out a crystallo-optical study of these preparations. The crystals of natural and synthetic uniaxial astrakhanites selected for optical investigation, were monoclinic, showed a grey interference colouring and a positive sign. The values obtained for the indices are very close to each other:

For synthetic astrakhanite	N_g=1.486;	N_m=1.486;	N_p=1.481
For natural astrakhanite from lake Khodzhamsayat	N_g=1.488;	N_m=1.487;	N_p=1.482
For natural astrakhanite from lake Bassinsk	N_g=1.486;	N_m=1.485;	N_p=1.481
For natural astrakhanite from Karabogaz gulf	N_g=1.486;	N_m=1.485;	N_p=1.482

It is of interest to note that the crystals of astrakhanites at our disposal essentially differ as to their shape from those described in literature. The crystals of astrakhanites studied by the author had a clearly pronounced lamellar habitus due to the highly developed face *c* (001) and the reduction of the face *p* (111) (Fig. 3). This lamellar character and the development of

face *c* (001) in astrakhanite crystals have been observed in laboratory experiments also by V. I. Nikolaev, D. I. Kuznetzov and G. B. Boky (1).

Under the microscope the uniform crystals of löweite show a yellow interference colouring. The refractive indices $N_g=1.491$, $N_p=1.473$ are nearly coinciding with the data by E. Larsen and G. Berman (7).

In the course of optical investigations of anhydrous astrakhanite its crystals were found to have a yellow interference colour and refractive indices $N_g=1.650$, $N_p=1.450$, different from those of vanthoffite and sodium and magnesium sulphates.

Thus, the above investigations give us ground to maintain that astrakhanite when heated suffers three conversions. The firts takes place at 105°, and is connected with a loss of two molecules of water by astrakhanite and a passing of the latter into löweite. Investigations showed löweite to contain two and not 2.5 water molecules, its chemical composition being expressed by the formula $Na_2SO_4.MgSO_4.2H_2O$.

The second conversion takes place at 215°. It is caused by the splitting off of hydrated water in the löweite and the formation of anhydrous astrakhanite. And lastly, the third one proceeds at 625°, being connected with a polymorphous conversion of the chemical compound $Na_2SO_4.MgSO_4$. For anhydrous astrakhanite the refractive index is determined as $N_g=1.650$, $N_p=1.450$. The temperature 670° corresponds to the fusion of anhydrous astrakhanite.

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Received
7. IV. 1939.

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Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1939. Volume XXIII, № 9

HYDROCHEMISTRY

OXIDIZING ACTIVITY AND PH OF BROWN SEDIMENTS OF THE BARENTZ SEA

By A. V. TROFIMOV

(*Communicated by W. I. Vernadsky, Member of the Academy, 2. V. 1939*)

The oxidizing activity of the sea bottom mud* and its reaction are the factors determining the rate with which the individual elements pass those stages of their cycle in the sea which take place at the bottom. These factors are also essential for the theory of the formation of iron-manganese concretions on the bottom of the sea. The biological formation of these concretions evidently occurs under the conditions of a more acid reaction in the sea bottom, favouring the removal of manganous and ferrous compounds which are the objects of the activity of iron bacteria ([2]).

As yet, however, no experimental data exist which might confirm the latter assumption. In general data on the pH and the oxidizing-reducing properties of sea bottom sediments are still only very scanty. Moore and Pirrie ([4,5]) carried out a few pH determinations of the slime water of sand deposits and obtained pH values which fluctuated between 7.6 and 8.2.

S. Brujewicz ([1]) carried out a number of «geographical» determinations of the pH and the oxidation-reduction potential (ORP) in the grey sediment from the neighbourhood of Spitzbergen and the reduced sediment of the Barentz Sea. The pH values he obtained for average sediment samples taken from the sea bottom varied within the limits 7.6—8.3.

Thus, the acidity of the averaged sediment samples exceeds that of the sea water above the bottom by no more than 0.5 pH.

While this problem was being investigated during the cruise on the s/s. «Knipovich» around Franz-Josef Land in September 1932, we were interested not so much in the average characteristics of the sediment samples as in the distribution along the profile of the sea bottom of the factors studied.

The section of the sea (see map) in which this investigation was carried out, is located in a region in which there spread markedly stratified brown sediments. The entire profile of the sea bottom here consists of three more or less sharply defined layers:

1) An upper, oxidized stratum (A) coloured brown by ferric compounds, from 0 to 15 cm thick;

2) A lower, reduced stratum (C), coloured dark and grey with different hues and extending downward throughout the whole depth which can be attained with the Eckman tube;

* According to the indication of W. I. Vernadsky throughout this article by «mud» are termed sea-sediments of no definite coarsnees (not necessarily fine).

3) An intermediate layer (B) which is transitional between the oxidized and the reduced strata and is usually from 0 to 10 cm thick.

Procedure. The oxidizing-reducing activity of the sea bottom was estimated in terms of two quantities: 1) the energy or the decomposition potential of quinhydrone in the sediment, and 2) the conventional oxidation-reduction potential (ORP or E'_H).

Both of these quantities were determined potentiometrically.

In order to determine the first quantity, an excess of quinhydrone was introduced into the mud sample and then a large excess of a powerful buffer system was added which consisted of a mixture of phthalic acid (1 part) and magnesium biphthalate (2 parts) ground together in a mortar (about 2 g per 10 g of mud). This buffer system, after it is mixed with the sediment, completely checks the reaction due to the latter and sets up in it a pH which is characteristic of the buffer itself ([3]). For the case of a medium having a salt concentration equal to that of sea water, the pH of this buffer is 3.5 and changes only slightly with temperature within the interval 6—15°.

The difference between the potential of a quinhydrone electrode dipping into this buffered mud and its normal value in the pure buffer is a measure of the oxidation or the reduction of the quinhydrone by the mud. This difference which we shall term the decomposition potential of quinhydrone, is equal to $\Delta = 29 \lg \frac{C_q}{C_h}$ mV at 18°C, where C_q and C_h are the concentrations of quinone and hydroquinone respectively in the mud at a state of redox equilibrium between the quinhydrone and the mud.

For mud which oxidizes quinhydrone, the values of Δ are positive, while for mud which reduces it they are negative. The limiting normal values of the decomposition potential Δ are determined by the formation of one of two oxidation-reduction systems in the sediment, namely, either: 1) a saturated mixture of quinhydrone and hydroquinone formed on the reduction of quinhydrone; for this case $\Delta = -85$ mv, or 2) a mixture of saturated solutions of quinone and quinhydrone formed on the oxidation of quinhydrone; for this case $\Delta = +52$ mv.

The conventional ORP was taken as the potential of a platinum electrode in the mud mixed with quinhydrone, and its value was calculated with respect to the normal hydrogen electrode. The difference between this conventional ORP and the true ORP may usually be assumed to be quite small in our case, because of the fact that the marine mud possesses a large potential store of sparingly soluble oxidation-reduction systems (Fe and Mn compounds) which are in equilibrium with the very weak, active, ionic, soluble part of these systems. Therefore in the majority of the sediments only an insignificant part of the potential O—R components of the mud will be used up in oxidation or reduction of the introduced quinhydrone to a state of equilibrium. The ORP will change comparatively little, and will become stable.

The pH was determined in two ways: 1) by employing the hydroquinhydrone method which is particularly suitable for reduced sediment strata and 2) by employing the quinhydrone method and introducing a correction for the decomposition of quinhydrone, i. e. taking into account the quantity ([3]).

Both of these methods yielded results which were in good agreement for the case of reduced mud. Measurements of 20 samples taken from the C stratum gave an average pH value of 8.24 and 8.30 when the first and the second method were employed, respectively. The greatest deviation between the data obtained with these methods was 0.25 pH.

The pH determinations of oxidized mud with the above methods were not always successful. Only when the values of Δ were not too high

(no greater than +40) could these determinations be made. If the oxidized sediment stratum A included dark manganese streaks, then oxidation of the quinhydrone by the sediment was so vigorous ($\Delta \geqslant 50$) that the sediment became noticeably alkaline and the pH values were markedly distorted. The electrode potentials here were not very stable.

All determinations were carried out on board the ship usually right after the sediment column was taken from the Eckman tube.

Results. A comparison of the upper layer (1—3 cm) of stratum A with the lower layer C (at a depth of 15—30 cm) was carried out at 21 stations. The upper layer of stratum A oxidized quinhydrone in all cases but one. The values of Δ varied from —5 to +54 mv. The lower stratum reduced quinhydrone in all cases, the values of Δ varying from —45 to —78 (mean $\Delta = -65$). Thus, the upper «brown silt» oxidizes quinhydrone quite vigorously. Even field soils, possessing a moisture content of about 50% which, consequently, is close to the moisture content of marine sediments,

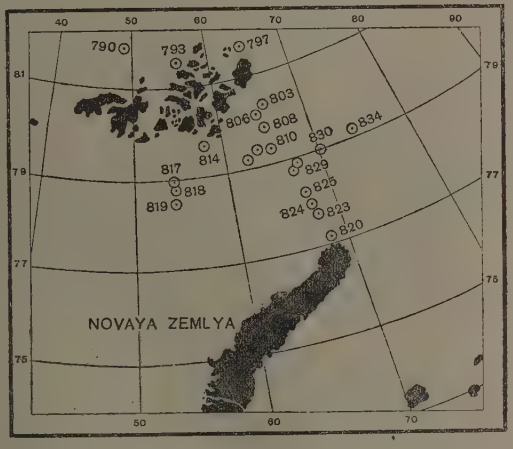

Fig. 1. Map. Location of the mud stations in the Barentz Sea (s/s. «Knipovich», September, 1932).

only very rarely possess such a high oxidizing activity. Thus, twelve soils from various parts of the USSR which we tested gave positive shifts of the quinhydrone potential Δ equal at the most to 20 mv. Only when the moisture content was 18—20%, the values of Δ for most active podzolized soils reached 30—35 mv, i. e. approached the brown silt in their oxidizing activity(3).

Between the upper stratum and the lower reduced layer, a difference in the values of Δ was observed in all cases which was equal on the average to 90—100 mv. The difference between the values of the conventional ORP is of the same order, E'_H being equal to 284 for the upper layer and to 188 mv for the lower layer.

As regards the pH, the upper layer is similar to the lower one: the mean pH value for the upper stratum is 8.20 and for the lower stratum 8.25. The extreme variations between the pH values for individual stations were 7.7—8.5.

At the last ten stations, 817—834, the change in Δ and in the ORP was traced in more detail—centimeter by centimeter from the surface of the bottom downward. At several stations this detailed measurement disclosed a comparatively marked acidity of the mud—up to a pH of 6.0 to 6.5—in the mud columns obtained from the intermediate layer (B)

of the bottom. This acidified layer is not very thick and is sharply tapering (with respect to its pH) both downwards and upwards. It is located in the region of the highest value of the ORP, in the zone of zero or weak reduction of quinhydrone. The most sharply defined acid streak was found at stations 818, 819 and 825—in the sea bottom possessing a thin oxidized stratum (3 cm thick) which sharply passes over into the lower layer C.

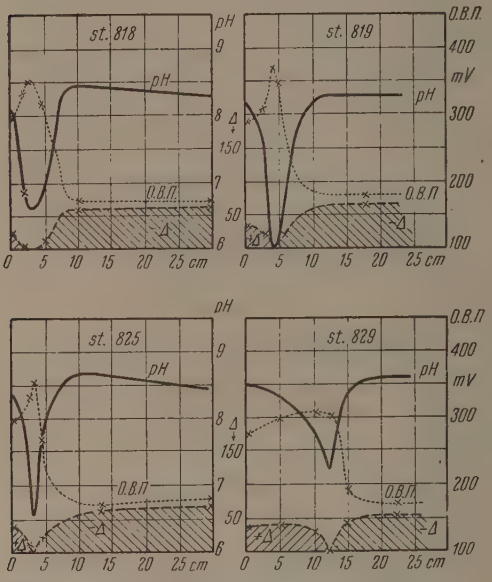

Fig. 2. Oxidation-reduction and pH curves of brown sediment possessing a clearly defined zone of acidification.

The most well defined oxidation-reduction and pH curves of the brown mud are given in Fig. 2.

The maximum gradient in the change in the pH of the sea bottom was observed at station 819; it is equal to 0.5 pH per cm. This phenomenon is very exceptional for a sea, since the change in pH throughout the whole 200—400 m layer of water above the bottom attained only 0.2—0.3 of a pH unit (8.2—8.3 at the sea surface and 8.0—8.1 at the bottom).

The presence of an acid streak in the profile of the brown mud is evidently due to an active process of oxidation which is caused by the dissolved oxygen of organic matter or manganous and ferrous carbonates present in the sea bottom. If the depth of the layer in which this process takes place is such that the removal of the CO_2 formed is hindered, then both of these processes should lead to the acidification of the sea bottom. Since the acidified stratum was not found at all the stations, this phenomenon can hardly be considered as stable. It probably bears a sporadic or perhaps a seasonal character, pulsating with a change in the ventilation near, and accumulation of organic matter at, the sea bottom.

When the reduced Barentz-Sea mud was oxidized in the laboratory with dissolved oxygen, a noticeable acidification of the mud was observed: after 30 days the pH of its upper layer decreased from 8.3 to 7.6, while at the same time the ORP increased from 170 to 250 mv and from 56 to 15 mv.

Received
28. III. 1939.

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Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1939. Volume XXIII, № 9

HYDROCHEMISTRY

RELATION BETWEEN THE CHLORINITY* AND THE DENSITY OF CASPIAN SEA WATER

By A. V. TROFIMOV

(Communicated by W. I. Vernadsky, Member of the Academy, 2. V. 1939)

A careful study has been made of the relationship between the chlorinity and the density for sea and ocean waters. This relationship written down in the form of formulae and tables (Knudsen's Hydrographic Tables), is widely used in hydrologic practice to calculate the density of sea water, the chlorine content being determined by titration.

The water of the Caspian Sea considerably differs in composition from ocean waters, and therefore one cannot employ the oceanographic tables to calculate the density of Caspian Sea water directly from the data on its chlorine content. In this case the salinity is usually calculated first by employing the «chlorine coefficient» which was found empirically [according to Lebedintzev (1) $k=2.38$], and then the density of ocean water having the same salinity is found with Knudsen's tables and gives the density sought for. This method of calculation fails to give correct absolute values of the density of Caspian Sea water. A value for the density is always obtained which is noticeably underestimated owing to the comparatively high content of relatively heavy salts—sulfate and magnesium salts—in Caspian Sea water. Its density is always larger than the density of ocean water of equal salinity.

However, not only the absolute, but also the comparative accuracy of such calculations is not great, since the temperature coefficient of the density for Caspian Sea water—due to the comparative deficiency of sodium chloride —is noticeably smaller than for ocean water.

In the Laboratory of Sea Chemistry of the Institute for Sea-Fisheries and Oceanography (VNIRO), an experimental investigation was undertaken in 1938 to discover a relation between the chlorinity and the density of Caspian Sea water, which would make it possible markedly to increase the accuracy in calculating the density of the Caspian Sea water by means of its chlorine number. Approximately the same degree of care and accuracy was observed in carrying out this investigation as was employed in the fundamental investigations to determine the sea water constants ([5–7]),

* The term «chlorinity» is widely used in the American literature and is equivalent to the usually employed terms «chlorine number» or «chlorine equivalent». The symbol σ is used throughout this paper to denote the conventional density of water. $\sigma_t=\sigma_t|_4=(D=1)\ 1\ 000$, where D is the specific gravity of water.

i. e., the absolute error of individual values of σ was no more than 0.01—0.02 and the maximum error of the chlorine data $^1/_{3000}$ of their values.

Water Samples. Altogether 15 samples of surface water were taken for analysis, of which 4 samples were taken from the open part of the southern Caspian, 8 samples from the northern Caspian in the region where fresh water is discharged by the Volga, and 3 samples from shallow water bays (from the Dead Kultzun and from the Bay of Balakhan). The samples had widely varying salt content, the range of variation being from 0.5 to 13‰ Cl.

As a general control of the entire analysis and in order to link it with the analyses of Knudsen-Sörensen, the chlorine content and the densities of three samples of Greenland Sea water were determined at the same time. This test led to concordant results—the values of σ for Greenland water, found experimentally and calculated according to Knudsen, differed on the average by 0.006.

Procedure. The density of the water was determined by the weight pycnometer method at two temperatures—0 and 25°C. The glass Knudsen pycnometers had a volume of about 100 cm³ and a capillary diameter of 0.8 mm. The meniscus in the capillaries could be read with an accuracy approaching 0,1 mg. After 3 hours in a snow bath and 1—2 hours in a bath at 25°C, the meniscuses became stationary. The temperature fluctuations of these baths did not exceed 0.01°C. Repeated weighings of the pycnometers showed departure lying within 0.1—0.2 mg. All results were reduced to vacuum. This was accomplished by employing the method which consists in a parallel weighing of a pycnometer tare having an accurately determined weight (reduced to vacuum). The volume of the pycnometer was repeatedly checked with bidistilled water. The average deviation of parallel determinations of $\sigma_{0°}$ and $\sigma_{25°}$ was 0.005.

Chlorine was determined by a weight titration of weighed samples of water. This determination was carried out according to the Volhardt method, somewhat altered: the precipitate of silver chloride was filtered off, then the filtrate was evaporated to a small volume and was titrated with a 0.01 *N* solution of rhodamine. Sodium arsenate was used as an external indicator to determine excess silver in case of Cl precipitation.

Standard water from the International Hydrographical Laboratory in Copenhagen served as the basis for calculating the chlorine numbers. The average difference between parallel chlorine determinations was 0.001‰.

Results. In order to find an empirical relation between the chlorinity and the density of Caspian water, the data obtained for σ_0 and Cl were treated after the method of least squares. It was found that this relation is quite well described by an equation of the 2nd order:

$$\sigma_0 = 0.036 + 2.090 \cdot Cl - 0.0063 \cdot Cl^2. \qquad (1)$$

The values for σ_0 calculated with this formula for the 14 samples of Caspian Sea water, deviated on the average from the experimental values by 0.006. Three of the 14 calculated values of σ_0 deviated from the experimental values by more than 0.01. Thus, in spite of the fact that the water of the northern Caspian is comparatively largely non-uniform, it was found possible to arrive at a formula that gives a relation which holds for the whole Caspian Sea.

Formula (1) cannot be applied to standard southern Caspian water which has been artificially diluted with bidistilled water; the relation between σ_0 and Cl for this case is expressed by another formula (2):

$$\sigma_0 = -0.132 + 2.101 \cdot Cl - 0.0025 \cdot Cl^2. \qquad (2)$$

This equation describes the limiting case of freshening of Caspian Sea water which does not occur in nature. The difference between the parameters of formulae (1) and (2) is mainly due to the fact that the water discharged by the Volga is not altogether free from salt.

From the density data for 25 and 0°C, the following relation between σ_0 and σ_{25}—was found:

$$\sigma_{25} = -2.808 + 0.933 \cdot \sigma_0 + 0.0013 \cdot \sigma_0^2. \qquad (3)$$

Both «Caspian» formulae (1 and 3) considerably differ with respect to their parameters from the similar formulae which were derived by Knudsen for ocean water. The density of Caspian Sea water calculated with our formula is much larger than the density of ocean water of equal chlorinity calculated with Knudsen's formula. In Table 1 there are given the corresponding differences (Δ_t') of the densities of both types of water.

Table 1

Difference between the Conventional Densities of Caspian Water and the Densities of Ocean Water of Equal Chlorinity at 0 and 25°C ($\Delta_t = \sigma_t$ Caspian—σ_t Ocean)

Cl ‰	Δ_0'	Δ_{25}'	$\Delta_0' - \Delta_{25}'$
1	0.72	0.69	0.03
2	1.33	1.25	0.08
3	1.92	1.82	0.10
4	2.50	2.38	0.12
5	**3.08**	**2.94**	**0.14**
6	3.64	3.47	0.17
7	4.19	4.00	0.19
8	4.74	4.52	0.22
9	5.27	5.03	0.24
10	5.79	5.53	0.26

In Table 2 there are given similar differences between the specific densities of Caspian Sea water and ocean water which have an equal salt content. These data are remarkable indeed since they afford a practical possibility for a sufficiently accurate calculation of σ_t for Caspian Sea water by making use of the chlorinity tests and Knudsen's tables.

Table 2

Increase in σ for Caspian Sea Water in Comparison with the Increase in σ for Ocean Water Having an Equal Salt Content at 0 and 25°C

S, ‰	Δ_0'	Δ_{25}'	$\Delta_{25}' - \Delta_0'$
5	0.34	0.36	0.02
6	0.41	0.43	0.02
7	0.47	0.49	0.02
8	0.53	0.55	0.02
9	**0.59**	**0.61**	**0.02**
10	0.65	0.67	0.02
11	0.70	0.73	0.03
12	0.75	0.78	0.03
13	0.81	0.84	0.03
14	**0.86**	**0.89**	**0.03**
15	0.91	0.94	0.03

From Table 2 it follows that the absolute values of the increments Δ' are only slightly dependent upon the temperature: the difference between Δ'_0 and Δ'_{25} is as low as 0.02—0.03 of a σ unit. One may assume that for all temperatures between 0 and 25°C the values of Δ' will lie between the values of Δ_0' and Δ'_{25}. Thus, if we interpolate the values for any temperature between 0 and 25° C, or even totally neglecting the temperature we do not make an error of more than 0.02 of a σ unit. Hence, the following method may be recommended to calculate σ_t for density Caspian Sea water.

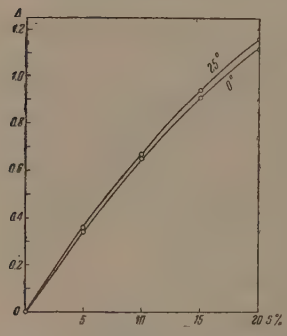

Fig. 1. The excess in σ-values of Caspian water as compared with oceanic water of equal salinity.

1) From the chlorine number, calculate the salinity S with the equation:

$$S = 0.14 + 2.36 \cdot Cl^*.$$

2) Having found the value of S, find the corresponding value of σ_t for ocean water in the oceanographical tables.

3) To this value of σ_t add the correction Δ' interpolated to the corresponding salinity and temperature (Table 2); we obtain the conventional density (σ_t) of Caspian Sea water with an absolute accuracy up to 0.02 which is sufficient for most hydrological problems.

In order to find the correction Δ' it is convenient to make use of the graph given in Fig. 1 which was constructed using the data of Table 2.

Received
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* This formula, similar to Brujewicz's([2]), is constructed «logically» on the basis of a knowledge of the chlorine coefficient and the average yearly salinity and alkalinity of the water of the Volga. In this equation the quantity S has the same meaning as Sörensen's S ([5]) in Knudsen's Tables [for details see([4])]. The data in Table 2 were also obtained by making use of this formula.

Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1939. Volume XXIII, № 9

CRYSTALLOGRAPHY

A CRYSTALLOGRAPHIC INVESTIGATION OF ARTIFICIAL MULLITE

By D. P. GRIGORJEV and I. I. SHAFRANOVSKY

(Communicated by V. E. Wernadsky, Member of the Academy, I. V. 1939)

Mullite crystals suitable for goniometric investigation are very rare. We know of only one such investigation of mullite crystals—an artificial product which was found protruding into pipes in the consolidated product of the mullite refractory (1).

A similar sample was also at our disposal. It originates from a flaw due to settling in the central part of a «brick» of mullite refractory which was obtained, it seems, employing the method described by E. S. Ginsberg (2).

The mullite appeared in the form of acicular, light grey crystals having a greenish tinge and a bright, glassy luster. The crystals were 2—3 mm in length and some tenths of a mm in thickness and partly grew together almost parallel in clusters.

The mineral was subjected to goniometric and optical crystal analysis.

The optical investigation consisted in a determination of the two refractive indices (by the immersion method) and the extinction angle of the crystals, and in stating the cleavage*. A comparison of the results of our measurements with the data of the American investigators (1) who had previously studied synthetic mullite is given in Table 1.

Table 1

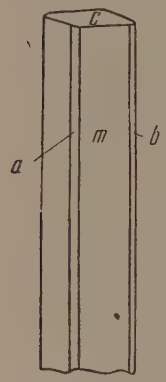
Investigator	Optical orientation	Refractive indices		Birefrigence	Cleavage	Axial angle $2V$
		N_g	N_p			
N. L. Bowen and J. W. Greig . . .	$N_g = c, N_p = b$	1.654	1.642	0.012	(010)	+45—50°
D. P. Grigorjev and I. I. Shafranovsky	$N_g = c, N_p = ?$	1.657 ±0.002	1.644 ±0.002	0.013	by lengthening	—

As can be seen in Table 1, all the properties approach those of pure artificial mullite. The somewhat larger values of the refractive indices of the described crystals are probably due to the presence of a slight amount of admixtures (1).

For the goniometric study we succeeded in selecting a few monocrystalline needles which were measured with a large bicircular Goldshmidt goniometer. During this investigation it was found that the faces belonging to the zone whose axis coincides with the direction of elongation give quite satisfactory light signals which can be detected with an accuracy approaching

* Under the microscope the mineral is completely transparent and colourless.

a few minutes. The mentioned zone consists of four well developed faces belonging to a pseudotetragonal rhombic prism with an almost square cross-section. This prism was mentioned by the previous investigators of mullite, Bowen and Greig, who gave it the symbol (110). In Fig. 1 its faces are indicated by the letter *m*. Furthermore, to the same zone (001) there belong the still slightly developed planes of the 1st and 2nd pinacoid—*a* (100) and *b* (010), blunting the edges of the prism. For mullite these forms were found for the first time.



All the measured crystals were broken off from one end.

Unfortunately, the dimensions of the second freely developing head are on account of the acicular facies of the crystals, so insignificant that we did not succeed in detecting any oblique faces whatsoever. Judging from the reflexion which occurred when the horizontal arm of the goniometer was at zero, the plane of the third pinacoid—*c*(011)—dominates here. It was pointed out above that the faces of the vertical zone (001) give quite clear light signals. For five crystals it was possible to obtain accurate values for the characteristic angles of the prism *m* (110). An average of 10 measurements gave the following values for the angles:

1) (110) : (110)=90°52′ (individual values varying from 90°22′ to 90°59′)
2) (110) : (110)=89°08′ (individual values varying from 89°01′ to 89°38′)

Bowen and Greig give the values 90°47′ and 89°13′ for the same angles*.

In Table 2 are given the spherical co-ordinates of all simple forms which we observed on mullite.

Table 2

Symbol	Size of side	Intensity of light signal	φ	ρ
m — 110	>	good	44°49′	90°
a — 100	≪	weak	90°	90°
b — 010	≪	weak	0°	90°
c — 001	≪	bright	∞	90°

In conclusion we wish to draw attention to the fact that the presence of such perfectly developed mullite crystals as those investigated once more confirms the complete independence of the compound $3Al_2O_3 \cdot 2SiO_2$ as a mineral.

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* Compare also with the data of L. L. Solodovnikova [3].

Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1939. Volume XXIII, № 9

HISTOLOGY

GENERAL CHARACTER OF THE GROWTH AND TRANSFORMATIONS OF THE ELEMENTS OF THE X-RAYED THYMUS OUTSIDE OF THE ORGANISM*

By SH. D. GALUSTIAN

(Communicated by I. I. Schmalhausen, Member of the Academy, 5. V. 1939)

The present work which forms part of a cycle of studies on the gl. thymus is devoted to a comparative study of the general character of the growth and transformations of the elements of normal thymic glands and a gland subjected to the action of weak doses of X-rays outside of the organism.

There exists a relatively small number of works concerning the cultivation of normal thymus outside of the organism ([1—5]).

Only one study (Chassovnikov) is devoted to the cultivation outside of the organism of fragments of an X-rayed gl. thymus. This author applied such powerful doses of X-rays that the viability of all tissue elements of the gland was greatly reduced. The explants in his experiments died off completely by the 20 th day of their life outside of the organism.

I used for my investigations the gl. thymus of young rabbits. One part of the rabbits was subjected to X-ray treatment applied to the sternal region and was killed 2 days after the experiment. Fragments of X-rayed as well as of normal (controls) thymus glands were cultivated separately in vitro according to the hanging drop method.

During the first 24 hours after explantation an emigration of lymphoid elements is observed in all gl. thymus cultures. Their number is relatively small in the X-rayed material. In the explants of normal thymus glands the emigration of lymphocytes shows a mass character. By the end of the first 24 hours they surround the explant in a wide ring.

The growth of the epithelium begins usually in the course of the second day. Its intensity and general character at the early stages do not differ noticeably in cultures from X-rayed and untreated glands. Alongside of the epithelium in the zone of growth appear also elements of the connective tissue which offer the usual picture.

The number of the emigrating lymphoid elements increases markedly during the next 2—3 days. In the control material they are distinguished by the pronounced amoeboid shape of their body. Only a relatively small part of the lymphocytes assumes a round form and suffers degeneration. In cultures from X-rayed material the mobility of the lymphocytes is less

* The experimental part of this work was made at the Department of Pathology of the Oncological Institute.

intense, many among them draw in their pseudopodia and begin to acquire a rounded shape, manifesting simultaneously increasing symptoms of degeneration.

The growth of the epithelium and connective tissue becomes visibly more active after the first transfer. Moreover, their growth in the test cultures is initially more intense than in the control ones.

Up to the 20—25th day of cultivation the general character of growth of the test and control cultures does not manifest any noticeable differences. Unlike most explants containing epithelial tissues, cultures from gl. thymus show exclusively an extensive growth of the epithelium in thick cellular masses and membranes. No phenomena of the epithelization of the explant's surface are observed. At the same time the intensely growing and proliferating in vitro epithelium of gl. thymus liquefies only in an insignificant measure the fibrin clot and, as a rule, fails to form any cavities of liquefaction.

The epithelial massifs advance gradually into the surrounding nutrient medium, and not infrequently the earlier sharp limit between the peripheric part of the explant and the growth zone is effaced and this peripheric part seems to dissolve in the latter.

The epithelial massifs and tracts of the zone of growth consist of closely adhering cells of a more or less polyhedric shape. The intercellular borders are usually clearly marked. The elements lie usually in two or three, sometimes even more layers. Mitoses are numerous.

In the control cultures the character of the epithelial membranes remains unchanged for a month or over and the rates of growth and proliferation do not sink.

As to the X-rayed material, in cultures from it the rate of growth of the epithelium loses its intensity beginning with approximately the 20—25th day after explantation. The size of the forming membranes becomes smaller after every transfer with excision of tissue. The numbers of mitotic figures decreases abruptly. Simultaneously the very character of the epithelial membranes suffers ever increasing alterations. The initially close distribution of the epithelial elements composing them becomes looser. Between the cells there appear—at first insignificant, in some places broadening—interspaces. The shape of the epithelial cells from polygonal gradually becomes less regular, there appear processi. At the same time the cells flatten out and become hypertrophied. Simultaneously ensues a more or less marked deformation of the nuclei. All these changes acquire gradually a pronounced degenerative character.

Contrary to the results obtained with powerful doses of X-rays the influence of the weak doses applied in the present work on the epithelium and connective tissue of gl. thymus becomes manifest only after a considerable lapse of time.

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Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1939. Volume XXIII, № 9

HISTOLOGY

IN VITRO CULTIVATION OF THE EPITHELIUM OF THE THYROID GLAND

By SH. D. GALUSTIAN

(Communicated by I. I. Schmalhausen, Member of the Academy, 5. V. 1939)

Attempts to cultivate bits of the thyroid gland outside of the organism have been made by many authors ([1],[2],[3],[6]). Ebeling succeeded to explant bits of the thyroid gland in vitro for a longer period than others, namely during 7 months. He describes even what he thinks to be the formation of new follicles with colloid in the zone of growth.

We were interested not in the fact itself of the possibility of cultivating bits of the thyroid gland, but rather in the qualitative peculiarities of the epidermal tissue of the thyroid gland which might be revealed under in vitro conditions. It was to be expected that the peculiar place and functional significance of this gland in the organism would be revealed in some definite manner in its epithelial components.

Bits of the thyroid gland of young rabbits and of pig embryos were used for cultivation outside of the organism. The cultures were conducted in the hanging drop.

Alterations begin to appear in the explant immediately after the culture has been prepared. The colloid begins to disappear, a mass crumbling off of epithelial cells into the cavity of the follicle takes place. Several hours after the explantation numerous mitotic figures make their appearance among the epithelial cells of the follicles. The epithelial cells of those follicles which lie at the periphery of the explant, begin to grow out into the surrounding medium and form epithelial membranes.

The growth of the epithelium is very intense, particularly after the first transfer, the epithelial membranes reaching considerable size and containing numerous mitoses.

Besides these membranes, there appear epithelial outgrowths in the shape of tongues or tracts which issue directly from the follicles into the surrounding medium. The epithelial tracts consist of closely adhering cells and attain considerable length. These tracts almost never liquefy fibrin and usually grow in the thick of the latter. The epithelial outgrowths produce branches which, uniting with the nearest tracts, interwine into a complicated network. Connective tissue and epithelial element frequently grow on one plane in the thick of fibrin. In such cases an impression is produced as though the epithelium «infiltrates» the connective tissue zone of growth.

We were unable to obtain genuine epithelization in explants from the thyroid of adult animals. Only in in vitro cultures of bits of the

thyroid of pig embryos is it possible to obtain the epithelization of such bits. In this case the epithelial cells are of a prismatic or cubic shape. In some cases the tegumental epithelium produces structures in several—three, four, rarer five-layers of epithelial elements.

Colloid is preserved in some of the follicles during the entire period of cultivation outside of the organism, i.e. up to 40 days.

The data of Ebeling concerning the eventual formation of follicles and colloid in in vitro cultures failed to be confirmed by our experiments. The obtaining of colloid formation under tissue culture conditions seems to me rather improbable with the methods at our disposal at the present time.

In studying the transformations of the epithelium of the thyroid gland under experimental conditions, one of the important problems is whether it is possible to discover in them the histioblastic potentialities characteristic of other representatives of this type of tissue. It was found that the epithelial elements of the thyroid show a marked lowering of the capacity to form differentiated tegumental structures, and that this capacity is shown only in explants prepared from embryos. Thus, the phenomenon of epithelization seems to weaken with the age of the initial material.

On the ground of the studied peculiarities of the growth of thyroid bits in vitro, the conclusion may be drawn that the epithelium of this gland represents a product of a specific evolutionary alteration of epidermal tissue. The notion that the thyroid gland has phylogenetically arisen from the endostylon of the ancestors, i. e. a groove-like deepening of the fore-intestine, checks well with our conclusions. The matting of the fore-intestine and its derivatives belong, as this has been repeatedly noted in the works of our laboratory ([4],[5],[7]), to the epidermal type.

The epithelium of the thyroid loses in a much greater measure—especially in adult animals—its tegumental properties and acquires the biological characteristics of a specific epidermal tissue of the inner medium of the organism.

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Received
5. V. 1939.

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Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1939. Volume XXIII, № 9

GENETICS

INHERITANCE OF ANTHOCYANINE COLORATION IN APPLE-TREE (A CONTRIBUTION TO THE PROBLEM OF BREEDING RED-FRUITED AND RED-FLESHED VARIETIES OF APPLES)

By P. D. TREBUSHENKO

(Communicated by N. I. Vavilov, Member of the Academy, 20. IV. 1939)

Red coloration both of the surface and of flesh of apples is a character which substantially increases the commercial value of the crop. It was paid great attention by S. V. Michurin in this country and by V. E. Hansen in the United States of America. It has been established that the pigment by which apples are coloured red belongs to the chemical group of anthocyanines.

So far as I am aware, no exact analysis of the inheritance of anthocyanine coloration in apple-tree has ever been made. There are indications by Hedrick and Wellington ([1],[3]), Auchter ([2]), Lantz ([4]), Crane and Lowrence ([5], [6]) to the effect that red coloration is dominant over the yellow one. The latter authors advance the assumption that «in the inheritance and distribution of the anthocyanine coloration may probably be involved several genes. The factors which heritably determine the anthocyanine coloration most likely are dominant»...

As the genetical constitution of the apple-tree is known to be complex (tetra-hexaploid), we decided to trace only the inheritance of the anthocyanine coloration, not that of its distribution. Most suitable for this purpose are forms which bear the anthocyanine coloration in all their organs, since, first, the factors determining the colour distribution are thus excluded from the analysis, and, secondly, it makes analysis possible at the stage of young seedlings and even on germinating plants, since the primary rootlets of the coloured plants also bear anthocyanine and possess a more or less pronounced pink coloration. The following forms (we shall term them for the sake of convenience coloured forms) bear the anthocyanine coloration in all organs at early age. *Malus Niedzwietzkiana* Dieck., *M. floribunda purpurea* Barbier, *M. purpurea aldenhamensis* Rehd., *M. purpurea Eleyi* Rehd., «*Melo japanese*», *M. purpurea Limoinei*. All these forms except the first and last one were used in our crosses. As a result of crossing 54 combinations were obtained in which one of the parents was coloured. Besides, we availed ourselves of two families derived from self-pollination of the coloured forms.

Upon examination of the whole material at our disposal, the following became clear: 1) Combinations in which partake coloured and colourless plants give both in their progeny. 2) Combinations in which both parents are colourless produce exclusively colourless progeny. 3) Forms bearing anthocyanine colour when self-pollinated produce both coloured and colourless progeny, the ratio not approximating 3 : 1.

It follows from the above: 1) That plants bearing anthocyanine coloration are heterozygous as to the factor for colour, since they produce a progeny consisting of the two kinds whether in the case of self- or cross-pollination. 2) That anthocyanine coloration is dependent on more than one pair of factors; otherwise from self-pollination we would obtain either uniform progeny (when the dominant factors are homozygous), or segregation approximating 3 : 1 (heterozygoty of the factors). 3) Forms and varieties bearing no anthocyanine coloration are either homozygous for the recessive factors, or (making allowance for the possibility of the coloration which is determined by several factors) heterozygous and homozygous for dominant factors which determine coloration only when interacting with other factors not contained in the plants in question in homozygous state.

Taking this for granted, we undertook a factorial analysis of the anthocyanine coloration.

In this work we were guided by the principle of analyzing crosses. We thought this principle to be the most efficient in the factorial analysis of such varieties which possess a complex genetical constitution, heterozygoty, self-sterility and late maturity. However, it is difficult to select recessive forms for the characters to be analyzed.

We have chosen as recessive forms such varieties in which the anthocyanine colour in the vegetative organs was scarcely pronounced under the influence of external conditions favourable for colour production. The variety «Pepinka Litovskaja» was included into this group. This variety was crossed with *Malus purpurea Eleyi* Rehd. This combination which was taken for analysis first, produced only 52 plants 14 of which were coloured and 38 colourless, a ratio which closely approximated 1 : 3. Such a ratio notably corresponds to the analyzing crossing in dihybrid segregation when the plant analyzed is heterozygous for the two factors. Taking this ratio for the starting point and having calculated the absolute values theoretically expected, we have obtained 13 for the coloured (q_0) and 39 for the colourless (q_1), as compared to the observed values: 14 coloured (p_0) and 38 colourless (p_1). The ratio of the latter values is 1.077 : 2.923 which is quite close to the theoretically expected one. The difference between the observed and expected ratios is $D=\pm 0.077$; the basic error (calculated from the formula for alternative variation) is $m=\pm 0.240$, $D/m=0.320$. From these data we arrive at a conclusion that our assumption as to the dihybrid nature of the anthocyanine coloration is correct. This is corroborated by other analyses, too. We have established in this way that the anthocyanine coloration which is present in all parts of the apple-tree depends on two dominant factors. One of them serves as the colourless base for the anthocyanine coloration; other factors cannot produce coloration without it. We have designated this factor through *F* (fundamentum), and its recessive allelomorph through *f*. The second factor which produces anthocyanine coloration only in the presence of and in co-operation with *F*, but fails to produce any coloration without *F*, we designate *M* (movimentum)*, its allelomorph is *m*. It was established at the same time that the analyzed form of *Malus purpurea Eleyi* Rehd. with respect to factors of anthocyanine coloration corresponds to the above-described structure heterozygous for the two factors, i. e. has the composition *FfMm*, while that of the variety Pepinka Litovskaja—is *ffmm*. It was shown by subsequent analyses that all the coloured forms used in our crosses are identical with respect to coloration to *M. purpurea Eleyi* Rehd., i. e. have the structure *FfMm*.

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* From moveo: setting into movement, producing influence, producing change.

As to the colourless forms, we found them to comprise: a) homozygous representatives recessive for the two factors of the structure *ffmm*; b) heterozygous for the factor *F* of the structure *Ffmm*; c) homozygous for the factor *F* of the structure *FFmm*.

We should emphasize the fact that in colourless forms analyzed by us in all cases only one identical factor was met with, and this was *F*.

All the above said is supported by the following data:

1. The fact that colourless individuals may be recessive for the two factors, i. e. *ffmm*, is supported by that a number of varieties and forms (see Nos .1—7 in the table) when crossed with coloured forms of the composition *FfMm*, produce in the progeny both coloured and colourless plants in the ratio approaching 1 : 3.

2. That they may be heterozygous for the one of the pairs of allelomorphic factors and have a composition *FfMm* or *ffMm* is verified by the fact that a number of individuals when crossed with coloured forms (see Nos. 7—12 in the table) produce a ratio approximating 3 : 5.

3. That they may be homozygous for one of the dominant factors, i. e. have the composition *FFmm* or *ffMM*, is shown by the fact that by crossing a series of forms and varieties with coloured ones a ratio is obtained in the progeny which approximates 1 : 1 (see Nos. 14—15 in the table).

That in non-coloured representations only identical dominant factors are present, i. e. that all of them have either *F* or *M*, so that among them either individuals with composition *Ffmm* and *FFmm*, or *ffMm* and *ffMM* are encountered, is shown by the fact that in none among 262 crosses of these representatives with one another were discovered in the progeny coloured plants which should be present in case factors *M* and *F* would meet (when distributed parallel among non-coloured forms and varieties). Lastly the fact that all the forms and varieties analyzed by us possess the dominant *F* is demonstrated by our observations on the reaction of the tissues of certain varieties of apples to injury by the San-Josè Scale.

The reaction consists in that the injured regions, the fruits in particular, acquire a more intense coloration (in case of varieties possessing coloured fruits). Fruits possessing no traces of anthocyanine coloration react according to the variety in two ways: in some varieties the anthocyanine coloration is similarly developed around the place of the prick; in the rest of varieties, no traces of anthocyanine coloration are developed around the prick.

This is accounted for by the fact that the saliva of the San-Josè Scale contains a substance which stimulates the anthocyanine formation in the presence of the leucobase of the anthocyanine coloration, i. e. a substance which has an action analogous to that of the factor *M*.

On the basis of these observations we arrived at a conclusion that the dominant factor of anthocyanine coloration which is rather often met with in non-coloured representatives is the factor *F*.

Having cleared up the genetic nature of the anthocyanine coloration and the factorial composition of the coloured forms, we have analyzed all the combinations available involving coloured forms. We excluded only those combinations which were represented by a small number of plants. In order to avoid subjectivism and give the real picture of the analysis, we have decided to give here all the combinations in which the plant number was not less than 30 (see table).

The following conclusions are drawn from the communicated above:

1. The anthocyanine coloration of all the organs of the apple-tree (particularly prominent in the young tissues) depends on two dominant factors, *F* and *M* .The factor *F* (fundamentum) is the leucobase of the anthocyanine coloration; other factors produce no coloration in its absence. The factor

No.	Combination No.	Name of the combination	Values					Ratios		D	m	$\frac{D}{m}$
			Empirical			Expected		Empirical	Expected			
			Total	Red	Green	Red	Green					
1	110	Pearmain winter gold × *M. purpurea Eleyi* Rehd.	38	7	31	9.50	28.50	0.737 : 3.263	1 : 3	0.263	0.281	0.936
2	154	*M. baccata* 525 A × *M. purpurea Eleyi* Rehd.	165	31	134	41.25	123.75	0.751 : 3.249	1 : 3	0.249	0.185	1.844
3	305	*M. prunifolia* Borkh × *M. purpurea Eleyi* Rehd.	45	10	35	11.25	33.75	0.889 ; 3.111	1 : 3	0.111	0.258	0.430
4	185	Pepinka Litovskaja × *M. purpurea Eleyi* Rehd.	52	14	38	13.00	39.00	1.077 : 2.923	1 : 3	0.077	0.240	0.321
5	153	*M. baccata* 525A × *M. purp. aldenhamensis* Rehd.	46	14	32	11.50	34.50	1.217 : 2.783	1 : 3	0.217	0.255	0.851
6	136	Calville-like × *M. purpurea Eleyi* Rehd.	49	15	34	12.25	36.75	1.224 : 2.776	1 : 3	0.224	0.247	0.907
7	122	Aport × *M. purpurea Eleyi Rehd.*	62	19	43	15.50	46.50	1.226 : 2.774	1 : 3	0.266	0.220	1.027
8	93	*M. baccata* 687A × *M. floribunda purp.* Bohlier	48	15	33	18.00	30.00	2.500 : 5.500	3 : 5	0.500	0.559	0.894
9	135	Cronsell transp. (?) × *M. purp. aldenhamensis* Rehd.	48	16	32	18,00	30.00	2.667 : 5.333	3 : 5	0.333	0.559	0.597
10	199	«Edel Roter» × *M. purpurea Eleyi.* Rehd.	83	29	54	31.12	51.88	2.795 : 5.205	3 : 5	0.205	0.425	0.482
11	302	*M. pumila praecox* C. K. Schn. × × *M. purpurea Eleyi* Rehd. . .	72	27	45	27.00	45.00	3.000 : 5.000	3 : 5	0.000	0.456	0.000
12	147	*M. purpurea Eleyi* Rehd. × Wagner	31	12	19	11.62	19.38	3.047 : 4.903	3 : 5	0.097	0.695	0.139
13	155	*M. purpurea Eleyi* Rehd. × *M. baccata*	74	36	38	37.00	37.00	0.973 : 1.027	1 : 1	0.027	0.116	0.233
14	66	*M. sylvestris* Mill × *M. floribunda purp.* Barbier	30	15	15	15.00	15.00	1.000 : 1.000	1 : 1	0.000	0.182	0.000
15	96	*M. floribunda purp.* Barbier × *M. sylvestris* Mill	43	23	20	21.50	21.50	1.069 : 0.931	1 : 1	0.069	0.152	0.454
16	76	«*Melo japanese*» (*self-pollination*). .	154	97	57	86.62	67.38	10.078 : 5.922	9 : 7	1.078	0.639	1.687

M (movimentum) causes formation of anthocyanine coloration in co-operation with the factor *F*. Without *F* the factor *M* produces no colour.

2. Forms *M. floribunda purpurea* Barbier, *M. purpurea Eleyi* Rehd., *M. purp. aldenhamensis* Rehd., «*Melo japanese*» have identical factorial composition *FfMm* with regard to anthocyanine colour.

3. Forms and varieties devoid of anthocyanine coloration may be: homozygous for the two recessive factors of the composition *ffmm*, heterozygous for the factor *F* of the composition *Ffmm*, homozygous for the factor *F* of the composition *FFmm*.

4. No forms or varieties of apple-trees devoid of coloration and possessing the factor *M* were discovered.

5. For preliminary factorial analysis for anthocyanine coloration in apple-tree against the background of the complete recessive a minimum number of seedlings may be as low as 32, i. e. twice the number of possible combinations of gametes.

The study of the nature and inheritance of the anthocyanine coloration deserves attention both from practical and theoretical sides. On the one hand, it makes it possible to produce homozygous forms for the two factors, of the composition *FFMM* (already obtained by us) which would produce 100% coloured plants in the first generation from crosses with any non-coloured form. Thus, the way is indicated to the solution of the problem of producing varieties possessing red-fleshy fruits with handsomely coloured surface. On the other hand, it is rather difficult to bring in accord such a simple segregation with the hypothesis of complex (tetra-hexaploid) genetic constitution of apple-tree ([7]). One may assume that factors *F* and *M* have arisen by mutation in comparatively recent period only in one of the 4 or 6 homologous chromosomes, whereas the remaining 3 or 5 bear only their recessive allelomorphs; however, wide distribution of the factor *F* in nearly every species and form of *Malus* speaks for its remote origin. It is, therefore, hardly probable that this factor is peculiar only to one chromosome which has from 4 to 6 homologues.

Besides, since non-coloured forms possessing the factor *M* were not discovered, the following suggestion may be made regarding the origin of the coloured forms: 1) either non-coloured forms possessing the factor *M* are uncommon both under natural conditions and in cultivation; then the conclusion becomes plausible that the coloured forms are due to mutation (arising of the factor *M*); 2) or they are limited to a definite area with special ecological and geographical conditions; then the occurrence of coloured forms may be explained by crossing of non-coloured forms possessing the factor *F* with non-coloured forms possessing the factor *M* in the area occupied by the latter.

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Received
3. V. 1939.

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Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1939. Volume XXIII, № 9

GENETICS

REDUCTION DIVISION IN INTERSPECIFIC HYBRIDS IN THE GENUS *BETA* L. II. CYTOLOGICAL EVIDENCE ON THE QUESTION ON EXPERIMENTAL SYNTHESIS OF THE SPECIES *BETA TRIGYNA* W. ET K. ($2n=54$)

By N. E. ZAIKOVSKAJA

(Communicated by N. I. Vavilov, Member of the Academy, 20. IV. 1939)

It was established in 1935 by means of ordinary cytological investigation of the seedling rootlets that one of the hybrids obtained by V. P. Zosimovich from the interspecific cross *Beta lomatogona* F. et M. ($2n=18$) × *Beta corolliflora* Zos.* ($2n=36$) had 54 chromosomes in its somatic cells instead of the normal 27 chromosomes. Such a chromosome complement could arise only as a result of duplication of the whole chromosome set of the hybrid and points, therefore, to its being an amphidiploid. V. P. Zosimovich has established that the amphidiploid which has originated from two wild species is morphologically more similar to a third wild species, *B. trigyna* W. et K. ($2n=54$), and suggested that he succeeded in reproducing artificially this species which, as he thought, had once arisen in an analogous way. This indication was kept in mind during all the cytological investigations and the amphidiploid was compared with the species *B. trigyna.*

In the present communication are presented the results of the study of reduction division in three interspecific hybrids—in the amphidiploid, in an analogous hybrid with single chromosome number, in the hybrid *B. lomatogona* × *B. trigyna* and in two polyploid species—*B. corolliflora* and *B. trigyna* (Table 1).

A great influence upon the course and the outcome of meiosis is produced by the process of chromosome conjugation which ends in the stage of diakinesis and may be most readily studied in beet just at this stage. In Table 1 are presented the results of counts of univalents, bivalents and higher associations at diakinesis in all the forms studied.

The process of chromosome conjugation is most regular in the tetraploid species *B. corolliflora* where there are no univalents at diakinesis. Higher associations (quadrivalents) are formed very rarely, although, following Zosimovich, the genom formula of this species should be *CCCC*, and M. I. Sirotina has established in its somatic set the presence of four chromosomes of each type.

In the species *B. trigyna*, in contrast to the data of Seitz, the full number of bivalents (27) was formed only once, whereas in the remaining 11 cases

* *B. corolliflora* Zos. (syn. *B. trigyna* f. *tetraploidea*)—a new species discovered by V. P. Zosimovich.

Table 1

Chromosome conjugation at diakinesis

Origin	Chromosome number	Formula of the genom	Quadrivalents	Trivalents	Bivalents	Univalents	Total number of elements	Number of cases
B. corolliflora Zos.	36	*CCCC*	2	—	14	—	16	1
	36	»	1	—	16	—	17	2
	36	»	—	—	18	—	18	5
B. trigyna W. et K.	54	*llCCCC*	—	—	27	—	27	1
	54	»	—	—	26	2	28	4
	54	»	—	—	25	4	29	3
	54	»	—	—	24	6	30	2
	54	»	—	—	23	8	31	1
	54	»	—	—	22	10	32	1
B. lomatogona F. et M. × *B. corolliflora* Zos.	27	*lCC*	—	—	9	9	18	4
	27	»	—	1	8	8	17	7
	27	»	—	2	7	7	16	2
	27	»	—	3	6	6	15	1
B. lomatogona F. et M. × *B. trigyna* W. et K.	36	*llCC*	—	—	18	—	18	1
	36	»	—	—	17	2	19	1
	36	»	—	—	16	4	20	2
	36	»	—	—	15	6	21	4
	36	»	—	—	14	8	22	4
	36	»	—	—	13	10	23	2
	36	»	—	—	12	12	24	1
B. lomatogona F. et M. × *B. corolliflora* Zos. (amphidiploid)	54	*llCCCC*	—	—	22	10	32	1
	54	»	—	—	21	12	33	2
	54	»	—	—	19	16	35	3
	54	»	—	—	18	18	36	1

the number of bivalents varied from 26 to 22 and that of univalents from 2 to 10, respectively.

In the amphidiploid the character of diakinesis is similar to that in *B. trigyna*, but chromosome conjugation in it is still weaker. The number of bivalents varies from 22 to 18 and that of univalents from 10 to 18, respectively.

In the hybrid $l_{18}C_{36}$ which possessed 27 somatic chromosomes, 18 chromosomes conjugated in all cases forming 9 bivalents; 9 univalents happened sometimes to lie all apart, sometimes 1—3 of them were attached to bivalents forming trivalent associations.

In the hybrid $l_{18}t_{54}$ possessing balanced somatic chromosome number (36), full number of bivalents was formed but once; in all the remaining 14 cases the number of bivalents varied from 17 to 12 and that of univalents from 2 to 12, respectively.

The further course of meiosis is also most regular in the species *B. corolliflora* where only isolated instances of single lagging chromosomes were observed; as a result of the two meiotic divisions in 97% of cases normal four nuclei were formed, all the four lie at the beginning in a common pollen mother cell (Table 2).

Table 2

Number of Daughter Nuclei and of Chromosomes Lagging in the Plasm after the Two Meiotic Divisions

Origin	2 nuclei	2 nuclei + chr. in the plasm	3 nuclei	3 nuclei + chr. in the plasm	4 nuclei	4 nuclei + chr. in the plasm	5 nuclei	5 nuclei + chr. in the plasm	6 nuclei	6 nuclei + chr. in the plasm	Total
B. corolliflora	—	—	—	—	97	3	—	—	—	—	100
B. trigyna	5	2	6	14	63	9	1	—	—	—	100
B. lomatogona × *B. corolliflora*	—	2	2	5	38	30	11	7	3	2	100
B. lomatogona × *B. trigyna*	—	1	1	5	72	18	2	—	1	—	100
Amphidiploid	1	2	1	10	55	26	4	1	—	—	100

No special term was ever applied to the stage in question. It lasts from the telophase of the 2nd division until the tetrad formation; during this period young daughter nuclei are formed of chromosome groups visible in the second telophase; they lie in beet all four in a common pollen mother cell; the latter divides them into four parts each of which contains a fully formed nucleus with a nucleolus. The results of meiosis in beet can be observed most clearly at this 4-nucleate stage.

In all the remaining forms the species *B. trigyna* included, the reduction division is characterized by a number of irregularities which are expressed in lagging of greater or lesser number of chromosomes, bridges, accessory small nuclei being formed and single chromosomes being thrown into the plasm. Sometimes the scattered chromosomes are collected after the first division into a common nucleus forming then «dyads»; aside from these, single chromosomes remain sometimes lying in the cytoplasm. Most often «dyads» were observed in *B. trigyna* (Table 2), less often in the amphidiploid and still less frequently in the hybrids $l_{18}C_{36}$ and $l_{18}t_{54}$. It is worth mentioning that, according to Table 2, the proportion of normal four-nucleate cells formed after the two meiotic divisions is higher in the hybrid $l_{18}t_{54}$ than in the species *B. trigyna*, which supports the view of V. P. Zosimovich as to the hybrid origin of this species. In the amphidiploid the outcome of meiosis is similar to that in *B. trigyna*. The hybrid $l_{18}C_{36}$ with 27 chromosomes exhibits the most disturbed process of reduction division and the lowest proportion of normal 4-nucleate cells.

In Table 3 are presented data on the quantity of pollen in anthers from mature flower-buds or from opening flowers.

The quantity and quality of mature pollen produced by the forms studied could be anticipated from the course of the meiotic process; however, aside from the mechanics of chromosome distribution one should take into account the qualitative side of the process, too. The number of regular 4-nucleate cells referred to above does not determine by itself the quantity of normal mature pollen in a given species. The hybrid $l_{18}C_{36}$, for instance, is not as much behind *B. trigyna*, or the amphidiploid in respect to the proportion of 4-nucleate cells as these latter are behind *B. corolliflora*; nevertheless, these species have as much pollen in mature anthers as *B. corolliflora*, and differ sharply from the hybrid $l_{18}C_{36}$. This should be accounted for by the fact that in the hybrid $l_{18}C_{36}$, as shown by the previous paper (2), the newly formed pollen cells contain most often from 12 to 14 chromosomes, out of which 9 are derived from divided bivalents and from 4 to 5 represent some of the univalents accidentally assembled at the one of the poles; they

Table 3

Amount of Mature Pollen in Anthers and Quality of the Pollen

Origin	Chromosome number	Quantity of pollen in mature anthers	Diameter of filled pollen in μ							Amount of degenerating pollen	Character of blooming
			40.0 — 35.1	35.0 — 30.1	30.0 — 25.1	25.0 — 20.1	20.0 — 15.1	15.0 — 10.1	10.0 — 5.1		
B. corolliflora .	36	much	+	+++	+++	++	+	+	+	isolated	open
B. trigyna . .	54	»	++	+++	++	+	+	++	+	very little	half-opened
B. lomatogona	18	»	—	+	++	+++	++	+	—	no	do
B. lomatogona × *B. corolliflora*	27	small amount	—	—	+	++	++	+	+	much	»
B. lomatogona × *B. trigyna*	36	much	+	+++	—+++	++	+	+	+	very little	»
Amphidiploid .	54	»	++	+++	++	+	+	++	+	very little	»

Note. In the present table + stands for «little», ++ for a medium amount, +++ for «much».

are inviable in spite of occasional formation of regular 4 nuclei. With regard to the quantity of pollen in mature anthers all the forms investigated could be divided into two groups; in the one was included the hybrid $l_{18}C_{36}$ which produces little mature pollen, in the other—the remainder. It is very probable that in the species *B. trigyna*, in the amphidiploid and in the hybrid $l_{18}t_{54}$ less pollen is formed than in *B. corolliflora*; however, it can be noticed neither by the naked eye, nor under the microscope. Pollen of the hybrid $l_{18}t_{54}$ does not differ qualitatively from that of *B. corolliflora*, whereas that of *B. trigyna* and of the amphidiploid is not as good and contains a considerable proportion of dwarfed grains.

Conclusions

1. The hypothesis of autotetraploid origin of *B. corolliflora* is supported by the regularity of the reduction division in pollen-mother cells in this species; one should emphasize the tendency toward formation of higher associations at diakinesis which, according to our evidence, is peculiar to polyploid forms and hybrids of beet in general.

2. The hypothesis of V. P. Zosimovich on the hybrid origin of *B. trigyna* is wholly supported by a number of irregularities in meiosis of this species, of the kind characteristic of hybrids.

3. The amphidiploid which arose spontaneously in the interspecific cross *B. lomatogona* × *B. corolliflora* and is morphologically similar to the species *B. trigyna*, proved similar to the latter in its cytological features, too: in chromosome number, in the course and outcome of meiosis, in the quantity and quality of mature pollen in ready flower buds. Formation of a lower number of bivalents in the amphidiploid is explained by its recent origin.

4. The data of Seitz on the regular formation at diakinèsis in *B. trigyna* of 27 bivalents failed to be checked in our material. The number of bivalents varied from 27 to 22 and that of univalents from 0 to 10.

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Received
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Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1939. Volume XXIII, № 9

PLANT PHYSIOLOGY

EFFECT OF TEMPERATURE UPON THE REVERSIBLE ACTIVITY OF INVERTASE IN FORAGE GRASSES AS DEPENDENT ON THEIR COLD AND HEAT RESISTANCE

By A. S. MOROSOV

(Communicated by A. A. Richter, Member of the Academy, 29. IV. 1939)

The problem of yield improvement in forage grasses and of their cultivation in dry regions and in far northern localities of the USSR is of prime importance and attracts therefore great attention. The true causes of death of plants remain nevertheless insufficiently studied. Modern theories which explain differences in the plant's reactions to low or high temperatures are based mainly on physical changes in the plasm. [(1–4) and others]. W. Lepeschkin (5) considers the death of the cell to be connected with coagulation of the plasm. However, as early as at the time of J. Sachs (8) cases were known when death of plants occurred long before physical changes in the plasm and thermal coagulation of proteins. This phenomenon may be explained, according to A. Kursanov, N. Krjukova and A. Morosov (6), by the disturbance of the normal fermentative activities of the cell while the latter is adapting its biochemical activities to new temperature conditions. Nobody has studied directly the effect of temperature upon the activities of ferments in living cells of meadow grasses. This is explained chiefly by the lack of technique for quantitative determination of ferments in uninjured tissues. But recently a method of vacuum infiltration was worked out by A. Kursanov(7) permitting of a quantitative determination of the synthesizing and hydrolizing activness of invertase in living plant cells. The work has been carried out at the All-Union Institute for Fodder-Research. As a material served hibernating leaves of awnless brome (*Bromus inermis*) and of English rye-grass (*Lolium perenne*) which were taken from under the snow; leaves of English rye-grass grown in a greenhouse were also studied. The synthesizing and hydrolizing activity of invertase was studied at a temperature from —5 to+50° C. The temperature intervals equalized 10° C. Depending on the osmotic pressure the cells were infiltrated with 0.1 and 0.2 molar solutions of saccharose for determination of hydrolysis; for determination of synthesis inverted sugar solution (composed of equal amounts of glucose and fructose) was used. After infiltration the plants were kept for 3 hours in chambers at a corresponding temperature. Experiments were arranged in 3 repetitions. The relations observed were the same in all cases. The results obtained are summarized in Table 1.

Table 1

Effect of Temperature upon the Reversible Action of Invertase in Hibernating Leaves of Awnless Brome *(Bromus inermis)* Taken from under the Snow

Experiment temperature, in degrees	Mg of invert during 1 hour per 1g of dry substance		Ratio synthesis / hydrolysis	Note
	Synthesis	Hydrolysis		
— 5	12.00	3.12	3.84	
0	4.25	2.75	1.57	
+10	5.30	14.50	0.36	
+20	5.37	7.37	0.71	
+30	1.30	9.00	0.14	
+40	1.10	14.70	0.07	
+50	0.0	25.00	0.0	Leaves died

The data obtained first of all testify to that temperature maxima in *Bromus inermis* are shifted toward low temperatures. This is especially conspicuous in plants which went through temperature «hardening». The hydrolizing action of invertase in leaves of *Bromus inermis* suggests three maxima: 1) at —5°, 2) at+10° and 3) at+50°. Synthetic action of invertase in hibernating leaves of *Bromus inermis* displays two temperature maxima: 1) at—5° and 2) at+20°. The ratio synthesis/hydrolysis makes it possible to judge of the trend of the processes in cells which suffer strong changes at various temperatures. At low temperatures synthesis significantly prevails over hydrolysis. This phenomenon was discovered in plants which were dug up from under the snow. At a temperature of —12° C the leaves were iced and there was neither synthesis nor hydrolysis. An entirely analogous situation was observed in hibernating leaves of English rye-grass *(Lolium perenne)*. The data obtained are presented in Table 2.

Table 2

Effect of Temperature upon the Reversible Action of Invertase in Hibernating Leaves of English Rye-grass *(Lolium perenne)* Taken from under the Snow

Experiment temperature, in degrees	Mg of invert during 1 hour per 1 g of dry substance		Ratio synthesis / hydrolysis	Note
	Synthesis	Hydrolysis		
— 5	7.90	4.30	1.83	
0	9.10	4.73	1.92	
+10	14.60	9.73	1.50	
+20	4.63	5.16	0.89	
+30	3.20	5.10	0.62	
+40	0.0	8.30	0.0	Leaves died
+50	0.0	15.12	0.0	Leaves died

On the other hand, in the leaves of *Lolium perenne* grown under greenhouse conditions the temperature maxima are shifted toward high temperatures. The hydrolizing action of invertase has two maxima: 1) at+30° and 2) at+50°. The synthesizing action of invertase has similarly two maxima: 1) at+20° and 2) at+30°. The data obtained are presented in Table 3.

Table 3

Effect of Temperature upon the Reversible Action of Invertase in Leaves of English Rye-Grass *(Lolium perenne)* Grown in a Greenhouse

Experiment temperature, in degrees	Mg of invert during 1 hour per 1 g of dry substance		Ratio synthesis / hydrolysis	Note
	Synthesis	Hydrolysis		
— 5	2.00	1.40	1.42	
0	1.70	1.10	1.54	
+10	2.34	1.96	1.19	
+20	4.65	3.07	1.51	
+30	3.82	3.82	1.00	
+40	9.82	7.00	0.45	
+50	0.0	9.63	0.0	Leaves died

The cold maximum is stronger expressed in those plants which passed the temperature «hardening». The ratio synthesis/hydrolysis also gives quite definite results which allow to judge on the adaptivity of the plant cell to high and low temperatures. The results of my experiments point to important deviations of the fermentative activity in meadow grasses, connected with the effect of high and low temperatures. It becomes quite manifest that ferments and biochemical processes which take place in plant tissues play a most important part in the process of dying of plants subjected to high and low temperatures.

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Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1939. Volume XXIII, № 9

PLANT PHYSIOLOGY

RELATIONSHIP BETWEEN THE REACTION OF MEDIUM AND THE ACTIVITY GROWTH SUBSTANCES

By J. V. RAKITIN and L. M. JARKOVAJA

(Communicated by A. N. Bach, Member of the Academy, 28. IV. 1939)

An array of data is available in the literature on the relation between the reaction of medium and the physiological activity of growth substances ([1–6]).

In the present paper a number of new data serve further to characterize the variation of growth reaction with the acidity of the medium.

Growth reaction was estimated according to Cholodny ([7]) whose method was modified by one of the present authors ([8]). The modification consisted in that the decapitated coleoptiles of *Avena* passed the test in a h o r i z o n t a l and not a vertical position. The drops of gelatin containing the substance tested were applied to the lower side of coleoptiles 2 mm from the top cut.

Preliminary experiments had shown that horizontally placed coleoptiles were more sensitive to substances promoting growth.

The coleoptiles of *Avena* were grown from the seed of variety «Pobeda». The seeds were allowed to germinate in Petri dishes in a room impermeable to light. The coleoptiles to be tested reached a length of 40 mm. They were prepared to the experiment under red light. The experiment was run in moist chambers at 23° C. Growth reaction intensity was always being recorded 1. 5 hour after the gelatin drop with the substance tested had been applied. The value to estimate it was the average degree of negative curvature measured in 10 coleoptiles.

The first set of tests involved the action of varying degrees of concentration of sulphuric, oxalic, citric and malic acids. The method of preparing the gelatin containing the acids was to compound aqueous solutions of 12% gelatin and of the respective acid in equal volumes while heating them. The medium thus prepared was cooled to 30° C before its drop had been applied to the coleoptiles. Conventionally the concentration of the acid was supposed to be reduced 2 times through blending with gelatin and if, for instance, it had been 2% in the initial solution it was assumed to have become 1% in the mixture.

The controls were given drops of 6% gelatin throughout the experiment. They failed to produce curvatures. The results of the experiment are presented in Table 1.

From the table it is evident that the effect of the acid was to change growth reaction considerably. The range of variation is easy to estimate as the control coleoptiles refused to bend within the time of the experiment.

Table 1

Acid tested	Content of acid in gelatin in %	Angle of curvature in degrees
Oxalic acid	0.5625	10.5 ± 0.70
	1.1250	30.3 ± 0.91
	2.2500	35.2 ± 0.84
Citric acid	0.6250	12.6 ± 1.51
	1.2500	32.3 ± 0.30
	2.5000	31.9 ± 0.25
Malic acid	0.6250	25.1 ± 0.61
	1.2500	40.4 ± 0.50
	2.5000	38.5 ± 0.75
Sulphuric acid	0.0625	15.4 ± 1.14
	0.1250	45.2 ± 1.01
	0.2500	16.1 ± 0.95

Further tests aimed to trace changes in growth reaction of coleoptiles with varying pH of buffer solutions (from 6.22 to 3.80). The latter were prepared according to Michaels by compounding 0.1 *n* acetic acid with 0.1 sodium acetate. Preparation of coleoptiles and the tests were conducted under the same conditions as in the tests with the acids. The results are summarized in Table 2.

Table 2

pH	Angle of curvature in degrees
6.22	0
5.60	8 ± 0.80
5.00	15 ± 0.75
4.40	30 ± 1.02
3.80	45 ± 0.91
Control	0

As it may be seen from these data, the intensity of growth reaction was growing with the concentration of hydrogen ions in the medium.

The last set of tests had for object to ascertain how the intensity of growth reaction is affected by growth substances acting against the background of varying concentration of hydrogen ions (Table 3).

An extract from oat kernels served as preparation of growth substances. To manufacture it 10 peeled and halved kernels were put in a mixture of 3 cm^3 of 96° ethyl alcohol and 5 cm^3 water. 34 hours later the extract was evaporated over a water bath to remove water entirely. The dry residue was dissolved in 2 cm^3 of hot water. With this solution obtained the preparation of growth substance was accomplished.

Table 3

pH	Angle of curvature in degrees	
	Control	Experiment
6.22	0	5 ± 0.98
5.00	10 ± 1.25	22 ± 1.40
3.80	35 ± 1.30	55 ± 1.89

The medium applied in drops to experimental coleoptiles presented a mixture of 1 volume of growth extract with 2 volumes of buffer solution and 1 volume of 12% gelatin. The controls were given drops of a medium consisting in 1 volume of water, 2 volumes of buffer solution and 1 volume of 12% gelatin.

This last set of tests affords unmistakable evidence that a given amount of growth substance varies within a wide range of activity according to the pH of the medium. The stronger the concentration of hydrogen ions, the more active are the growth substances.

Another conclusion to be drawn from Table 3 is that it will often suffice to change the reaction of the medium for bringing about more intensive growth as could be achieved by adding growth substances without changing the concentration of hydrogen ions. While the addition of growth substances at pH=6.22 produced an angle of curvature equal to $5° \pm 0.98$, the mere shift in the reaction of the medium to pH=5.00 made the curvature increase up to $10° \pm 1.25$, and if continued to pH=3.80 the increase of the curvature reached an angle of $35° \pm 1.30$.

Literary and experimental data referred to in the present paper seem to justify the assumption that by changing the concentration of hydrogen ions in the medium we may in a number of cases (collus formation, rooting of cuttings, artificial parthenocarpy and others) not only make the use of synthetic growth substances (heteroauxin, etc.) more effective, but sometimes even entirely dispense with special growth promoting preparations.

In conclusion some notes will not be amiss concerning the influence which the reaction of the medium exerts upon the results of a determination of the physiological activity of growth substances.

The fact that this activity is strongly dependent upon the pH of the medium involves the impossibility to get a proper notion of the physiological activity of various growth substance preparations, unless there is no change in the reaction of the medium during the test of all of them.

In this connexion it should be noted that an exact determination of growth substances is not to be achieved if the method applied is based on their diffusion from the subject tested into agar, whereupon the growth reaction of coleoptiles to the substances diffusing from agar is recorded.

Indeed, the growth substances (auxins) do not make alone their way from the subject tested (vegetable tissue) to agar and further on to the decapitated coleoptiles, but they are certainly attended by the acids which have been shown to change the activity of growth substances appreciably. If the subjects tested for the content of growth substances are different physiologically, their acidity as a rule will be different, too. This difference in acidity will be imparted to agar blocks and cause them to produce a variable influence upon the growth reaction of coleoptiles. It stands to reason that with the ordinary diffusion method applied we will meet cases where the agar blocks in contact with the test subjects having a low concentration of auxins, but a high acidity, will accelerate growth more considerably than do those touching the subjects in which a high auxin concentration is attended by a low acidity.

And besides the auxins and acids there are of course still other substances in the vegetable tissue which may affect the growth reaction of coleoptiles in some other way if the method of agar blocks is made use of.

Therefore we should not forget, while having recourse to this method, that the growth reaction observed in coleoptiles may have been produced not by the auxins alone, but by several other substances, too, among which the first place belongs to the acids.

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Received
29. IV. 1939.

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Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1939. Volume XXIII, № 9

ANIMAL PHYSIOLOGY

FURTHER INVESTIGATIONS ON THE MECHANISM OF THE FORMATION AND DESTRUCTION OF THE CHEMICAL FACTORS OF NERVOUS STIMULATION

By CH. S. KOSHTOJANZ, Corresponding Member of the Academy, and R. L. MITROPOLITANSKAJA

In an earlier communication published in this review in March 1938, one of us (1) described the results of experimental investigations which supplied grounds for advancing a hypothesis concerning the formation of the so-called chemical transmitters of nervous excitation, in particular of acetylcholine-like substances. In this communication the supposition was expressed that the formation of acetylcholine and acetylcholine-like substances is closely bound with the processes of cell metabolism, with carbohydrate metabolism in particular. It was demonstrated that fluoride blockade of the decomposition of carbohydrates coincides with the blocking of the transmitting of excitation from the nerve to the muscle, both the cardiac or the skeletal. Such an effect of the action of fluoride (sodium fluoride) was referred to the delay which it provoked in the formation of a number of products of carbohydrate decomposition (pyroracemic acid, acetic acid), able to form physiologically active esters of choline, acetylcholine in particular. In the course of the year which has been passed since, numerous experimental data both physiological and biochemical have accumulated in the literature which corroborate the correctness of the supposition advanced at the time. Thus, the Cambridge biochemists Mann, Tennenbaum and Quastel (2) supply on the ground of special biochemical researches a scheme of the formation of acetylcholine in the nervous cell from choline and decomposition products of carbohydrates, a scheme which confirms in its main lines the hypothesis advanced by us. A study issued by the laboratory of the eminent electrophysiologist Bremer (3), shows that the character of the electric potential of the cerebral cortex cells is strongly affected by the action of sodium fluoride, the changes arising in the electric potential being ascribed by the author to the primary action of sodium fluoride on the normal course of the carbohydrate metabolism of the nervous cells.

Developing further the work in this direction in our laboratory, we investigated a series of substances which exert a specific action on the chemodynamics of the cell, of the nervous cell in particular, with the purpose of inquiring deeper into the chemical aspect of the relations between the formation of substances involved in the nervous process, and the fundamental processes of cellular metabolism. Besides sodium fluoride we tested glyceraldehyde, sodium maleinate, glutamic acid. The results of the investigation of the action of these substances on the process of transmission of impulses from the motor

nerve to the striate muscle are described in a special communication by A. M. Rjabinovskaja in the present issue of this periodical. In the present note we report the results of our further investigations concerning the transmission of excitation from the nerve to the cardiac muscle under conditions of carbohydrate metabolism, inhibited by the effect of glyceraldehyde. The researches just cited, similarly to our earlier investigations, concern the mechanism of the formation of one of the known groups of substances participating in the nervous process—of acetylcholine-like substances. Of late we have set ourselves the task to approach also the problem of the transfer of impulses by means of the formation of adrenaline-like substances, which is the case in the stimulation of the sympathetic nervous system. Heirman ([4]) succeeded in demonstrating that by acting on the heart with such oxidases as phenolase or thyrosinase, one can eliminate the effect of the action of the stimulation of n. sympathicus on the heart, apparently owing to the oxidations of the adrenaline-like substance which is formed in the course of this

Fig. 1. The effect of glyceraldehyde (M/50 solution) on the response of the isolated frog heart to electric stimulation of the mixed nerve-trunk vago-sympathici. I—response of normal heart; II—response of heart after treatment with glyceraldehyde (the effect of the action of n. vagus is absent); III—response of heart to acetylcholine 10^{-6} (designated by pointer) after treatment with glyceraldehyde; the acetylcholine effect is preserved. the preceding stimulation of the nerve produces no effect; IV—restoration of the action of nerve stimulation after the washing off of glyceraldehyde.

excitation, and to its transformation into an inactive form. Particular importance in processes of oxidation is known to be ascribed now to cupric protein compounds and, in particular, it has been shown that certain oxidases, as e. g. the polyphenol-oxidase, are complex compounds of copper and protein. Proceeding from these data, we set ourselves the task to investigate the action of minimum dilutions of cupric salts on the heart of the frog and the effect of this substance on the mediation of impulses from n. sympathicus to the heart. Theoretically one might anticipate that copper in this case would act similarly to the phenolase in Heirman's experiments, i. e. eliminate the effect of the action of n. sympathicus by oxidizing the adrenaline-like substances.

The experiments were conducted on the heart of a frog with electric stimulation of the mixed nerve trunk n. vago-sympathici. It was found that after the treatment of the heart with glyceraldehyde (M/50 solution) the typical response (stopping) of the heart to the excitation of n. vagus is absent. In our opinion, this effect is accounted for by the fact that the disturbance of the normal course of carbohydrate metabolism due to the glyceraldehyde action led to the disturbance of the normal process of formation of acetylcholine-like substances (Fig. 1). As may be seen in Fig. 1, IV, the action of glyceraldehyde is reversible. After glyceraldehyde has been washed off, the action of the stimulation of n. vagus is restored. Material is the fact that while electric stimulation of n. vagus remains inefficient within the period of action of glyceraldehyde, acetylcholine introduced from outside when acting on the heart provokes a typical effect (Fig. 1, III). This last circumstance shows the protoplasm of the heart muscle cell being able to respond

to acetylcholine under conditions of a simultaneous action of glyceraldehyde. It follows hence that the absence of the effect of the stimulation of n. vagus may be put down to the disturbance of the process of formation or liberation of acetylcholine-like substances.

As should be anticipated, when the mixed nerve trunk vago-sympathici is subjected to stimulation after the heart has been acted upon with glyceraldehyde, the action of n. vagus is eliminated and the prevalent action of n. sympathicus becomes evident (Fig. 1, II). Having obtained this prevalent sympathetic effect after preliminary elimination of the action of

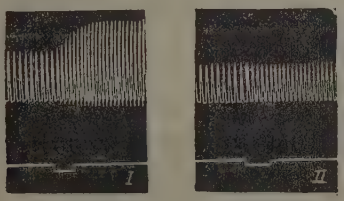

Fig. 2. The effect of $CuCl_2$ (dilution 10^{-5}) on the response of the isolated frog heart to electric stimulation of the mixed nerve-trunk vago-sympathici. I—sympathetic effect after the elimination of the action of n. vagus by means of glyceraldehyde; II—elimination of sympathetic effect after the action of cupric solution.

n. vagus by means of glyceraldehyde, we set ourselves the task to eliminate the action of n. sympathicus by means of copper, proceeding from the above theoretical considerations. Indeed, the action of $CuCl_2$ solution (dilution 10^{-5}) proved to eliminate the effect of n. sympathicus (Fig. 2).

The above results form part of the researches conducted by us on the question of the mechanism of formation of chemical factors of the nervous process and on the possibility of eliminating the effect of one or other nerve (of the vegetative nerves in particular) by acting upon definite sides of metabolism both of nervous and of other cells.

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Received
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Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1939. Volume XXIII, № 9

ANIMAL PHYSIOLOGY

INFLUENCE OF SODIUM FLUORIDE, SODIUM MALEINATE, GLUTAMIC ACID AND GLYCERALDEHYDE ON MUSCULAR ACTIVITY

By A. M. RJABINOVSKAJA

(Communicated by A. N. Bach, Member of the Academy, 5. V. 1939)

At the present time the study of the problem of transmission of stimulation has entered a new phase: from the stage of the discovery and constatation of chemical mediators the attention has turned to how these substances have arisen. A number of studies have shown that in the incubated surviving nervous tissue of mammals there arise acethylcholine-like substances ([10],[11]). The question regarding the mechanism of the formation of these substances which arose in connexion herewith, forms the subject of a series of studies and hypotheses. Thus, Quastel and his collaborators (1938) have demonstrated that acethylcholine is formed in vitro in incubated cerebral tissue, and have supplied an eventual scheme of the formation of acethylcholine from its precursor. Dickshit ([2]) investigated a number of tissues and organs in respect of the possibility of acethylcholine formation in them.

Ch. S. Koshtojanz ([5]) proceeding from the fact that some intermediate products of tissue metabolism are able to form esters of choline, particularly such esters which possess a high physiological activity (such as e. g. piruvate-choline, acethylcholine), advanced the idea that the formation of chemical mediators is bound with the general chemodynamic processes of the organ in question or of the entire organism. To such metabolic products able to form physiologically active esters of choline, belong the pyroracemic and acetic acids and acetaldehyde.

The elimination of the processes which lead to the formatiom of these substances ought, according to the opinion of Ch. S. Koshtojanz, result in a disturbance of the formation of chemical mediators.

Quastel et al. (1938) indicate a series of conditions affecting the formation of acethylcholine in cerebral tissue in vitro. The following of these conditions should be noted: a necessary condition of acethylcholine formation is the presence of oxygen and of glucose. Similar to that of glucose is the influence of lactate and piruvate. Contrariwise, a number of compounds, such as sodium fluoride, glyceraldehyde and some others, hinder the formation of acethylcholine. It is known that these substances generally affect the tissue metabolism, especially carbohydrate metabolism. Fluoride and glyceraldehyde, according to a number of authors, inhibit anaerobic and aerobic glycolysis: The salts of maleinic and glutamic acids affect glycolysis in a similar manner.

According to the observations of numerous authors, the action of fluoride is analogous to the action of mono-iodine-acetic acid, since both compounds prevent glycolysis. Yet Meyerhof ([8]) has shown the mechanism of their action to be different: mono-iodine-acetic acid affects an earlier stage of glycolysis than fluoride. Therefore certain intermediate stages of the process which proceed in the presence of sodium fluoride are absent in that of mono-iodine-acetic acid. As to the effect of sodium fluoride, this, according to Meyerhof, consists in that it hinders the conversion of phosphoglycerol acid into phosphopyroracemic one.

Besides fluoride, glycolysis is affected also by glutamic acid. After Weil-Malherbe, data ([16]), it inhibits anaerobic glycolysis in cerebral tissue, raising simultaneously aerobic glycolysis. This checks perfectly with the data by Dixon ([3]) who has shown that glutamic acid hinders Pasteur's effect in the cerebral tissue without, however, decreasing respiration. Glutamic acid offers great interest since, unlike most amino-acids, it is oxidized in the organs the metabolism of which, according to the available data, is realized chiefly at the expense of carbohydrates and which are completely inert in regard to most other amino-acids. And this points to the existence of some relation between glutamic acid and carbohydrate metabolism.

In a similar way the metabolism of cerebral tissue is influenced by maleinic acid: it also inhibits Pasteur's effect, decreases anaerobic glycolysis and raises the aerobic one. The mechanism of its action on fermentative processes has been studied in sufficient detail. Morgan and Friedman ([9]) have shown that maleinic acid hinders the fermentative processes determined by sulphhydril compounds, or decreases the rate of these processes.

Maleinic acid reacts with the sulphhydril group of glutathione forming with it a stable compound, inhibiting in this way the anaerobic glycolysis.

As it is known, glutathione, acting upon the muscle of the mammal, increases the formation of lactic acid in it about 20 times. A similar action of glutathione may be observed also in other tissues: brain, liver, heart muscle and uterus muscle.

Thunberg ([12]) has shown that maleinic acid hinders the respiration of the muscle, too. The same was found by Weil-Malherbe ([16]): unlike a great number of other organic acids, maleinic acid inhibits respiration in the brain section, depressing it in different tests to 10—50% of the normal value. Simultaneously with the sinking of anaerobic glycolysis, the aerobic glycolysis is raised reaching the level of the anaerobic one.

Finally, as regards glyceraldehyde, it has been found to hinder glycolysis both in normal tissues (Sülmann, 1938) and in the cells of carcinomas ([7]).

On the ground of the data cited above and hypotheses concerning the conditions of the formation of acethylcholine, the question arises as to the character of the influence of these substances on muscular activity.

It should, however, be noted that the hindering of glycolysis in the muscle leads to a disturbance of the energy-transformations and, therefore, cannot be used as a direct criterion of the disturbance of acethylcholine formation and of the transmission of excitation bound with it. Nevertheless, the effect of these substances on muscular activity presents a certain interest in the light of the data cited above.

P r o c e d u r e. The experiments were made on a nerve-muscular preparation (ischiadicus-gastrocnemius) and on m. sartorius of the frog. From every pair of muscles one was left in Ringer solution for control, while the other was placed for varying time intervals (15—90 min.) into the solution of one of the substances enumerated. After this the isotonic contractions of all the muscles—experiment and control ones alike—in response to single disjunction-induction stimulations was registered. The stimulations used

were either direct (in the case of both muscles), or indirect (in the case of gastrocnemius). The threshold and the maximum of irritation were determined both before and after the action of the substances enumerated above.

Results. 1. Sodium fluoride in a concentration of M/50 acting for one hour produces in the nerve-muscular preparation a reduction of the amplitude and a rise of the excitability at direct stimulation; indirect excitability is lacking (Fig. 1).

2. Sodium maleinate in a concentration of M/400 in the presence of phosphate buffer, pH=7.2—7.3, provokes a lowering of direct excitability in both muscles and of the indirect

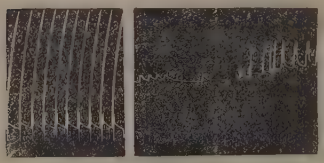

Fig. 1. Left—normal m. gastrocnemius, indirect irritation. Right—the same muscle after one hour exposition to NaF, indirect irritation (left part of curve) and direct irritation (right part).

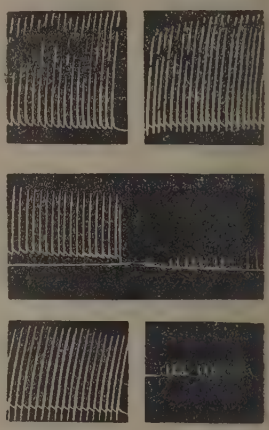

Fig. 2. Above—normal frog gastrocnemius. Left—direct irritation, right—indirect, middle—the same after one hour exposition to sodium maleinate. Below—m. sartorius, direct irritation, left—normal muscle, right—the same after a 5 min. exposition to sodium maleinate.

one in m. gastrocnemius (Fig. 2). As should be anticipated in respect of direct excitability the effect is manifested in m. sartorius earlier than in m. gastrocnemius.

3. Sodium glutaminate in a concentration of M/100 acting on the nervemuscular frog preparation during one hour eliminates completely, or

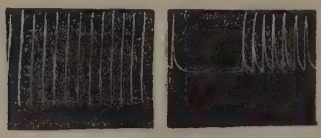

Fig. 3. Left—normal gastrocnemius, indirect irritation. Right—indirect and direct irritation of the same muscle after an 1,5 hour exposition to the action of sodium glutaminate. Below—the same after an hour exposition to glyceraldehyde.

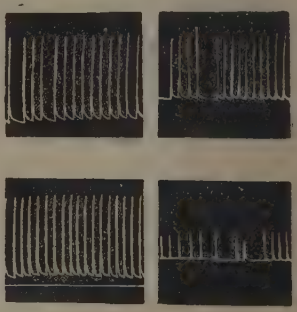

Fig. 4. Above—normal gastrocnemius, direct and indirect irritation. Below—the same after an 1 hour exposition to glyceraldehyde.

almost completely the indirect excitability (Fig. 3). Direct excitability is also lowered, but not to such a degree as in the case of sodium fluoride.

4. Glyceraldehyde (M/50) provokes in the nerve-muscular preparation a decrease of both the direct and the indirect excitability (Fig. 4).

The results obtained though giving no direct answer to the question concerning the conditions of formation or liberation of acethylcholine, never-

theless point to the compounds tested to affect both the direct and the indirect excitability of skeletal muscles. Acethylcholine may possibly stand in some relation to those intense processes which take place in a normal muscle and which are influenced by the compounds used in the described experiments, the more so as acethylcholine is known to hinder the breaking up of phosphocreatine ([1]). On the other hand, we know creatinin in weak concentrations to intensify the acetylcholine effect on m. rectus abdominis of the frog ([4]).

These data once more corroborate the statement advanced by Ch. S. Koshtojanz to the effect that acetylcholine is not only a mediator of the processes of excitation, but also participates in the general chemodynamic processes of the organ or organism.

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Received
21. IV. 1939.

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Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1939. Volume XXIII, № 9

ENTWICKLUNGSMECHANIK

DIE EINWIRKUNG DES NERVENSYSTEMS AUF DIE REGENERATION DER GLIEDMASSEN BEIM AXOLOTL

Von J. SCHAXEL und G. SCHNEIDER

(Vorgelegt von I. I. Schmalhausen, Mitglied der Academie, d. 5. V. 1939)

Trotz zahlreicher Untersuchungen, die in den letzten 40 Jahren durchgeführt worden sind, besteht keine Einigkeit darüber, in welcher Weise das Nervensystem an der Regeneration Anteil nimmt. Wir verweisen auf die zusammenfassenden Darstellungen von E. Korschelt ([3]), M. Abeloos ([1]), S. R. Detwiler ([2]).

Der allgemeine Grund der widerspruchsvollen Angaben der Autoren besteht in methodologischen Mängeln. Es ist nicht eindeutig klar gemacht:

1. Was die Regeneration im Entwicklungsablauf eigentlich ist.
2. Welche Unterschiede zwischen den regenerativen Vorgängen bei den Wirbellosen und Wirbeltieren bestehen.
3. Welche Stadien der Regenerationsprozess selbst durchläuft.
4. Welche Wirkungen das Nervensystem bei den Wirbeltieren auf die Formbildung haben kann: a) stimulierende und trophische, oder b) spezifisch morphogenetische, oder c) eine Kombination von beiden.
5. Welchen Anteil bei den etwaigen morphogenetischen Wirkungen des Nervensystems der Wirbeltiere das sympathische und das cerebrospinale Nervensystem und bei diesem die sensible und die motorische Innervation hat.

Dazu kommen je nach dem Objekt, mit dem experimentiert wird, einige besondere Fehlerquellen. Im Falle der Regeneration von Extremitäten urodeler Amphibien sind es hauptsächlich folgende:

6. Die Unkenntnis der morphologischen und physiologischen Variationsbreite der Innervation sowohl nach ungestörter Entwicklung wie nach ein- oder mehrmaliger Regeneration.
7. Die Unkenntnis der Regeneration des Nervensystems selbst, besonders die der peripherischen Nerven.

Diese Mängel und Fehler haben wir bei unseren Untersuchungen ausgeschlossen. Was die Regeneration ist und welche Stadien sie durchläuft, zeigten wir in früheren Untersuchungen ([4–8]). Die Regeneration ist nicht Wiedererzeugung eines verloren gegangenen Körperteils, sondern im Organismus kommt es nach Entfernung eines Teiles im Fortgang der Bildungsvorgänge zur Herstellung eines atypischen Gebildes vom Restbestand aus. Nicht, was fehlt, entscheidet, sondern was vorhanden ist. Bei der Regeneration sind die Leistungen der Zellen, Gewebe und ihrer Produkte im Einzelnen dieselben wie in der Ontogenese überhaupt, aber sie verlaufen räumlich und zeitlich unter anderen Bedingungen wie die ersten Bildungen beim Aufbau

des Organismus. Deshalb ist der Regenerationsprozess atypisch und alle Regenerate sind atypische Gebilde, Ersatzgebilde (Ausschluss des Fehlers 1).

Die Determination der Regeneration geht so vor sich, dass ein indifferenter Ausgangszustand in Abhängigkeit vom regenerierenden Restbestand in einen einsinning bestimmten Endzustand übergeht. Die Knospen sind noch unbestimmte, vorbereitende Stadien, die äusserlich ungeformt sind, histologisch aus gleichartigem indifferentem Material bestehen und sich in Abhängigkeit eines übergeordneten Zusammenhanges entwickeln. Die Anlagen gewinnen bereits äussere Form, ihr Material weist unterscheidbare Schichtungen auf und ihre weitere Entwicklung vollzieht sich als Entfaltung im engeren Sinne zum bleibenden Endgebilde (Determination in successiven Akten) (Ausschluss des Fehlers 3).

In dieser kurzen Mitteillung übertragen wir keine an Wirbellosen gewonnenen Ergebnisse auf die Wirbeltiere, da erst nach der völligen Klarstellung der Vorgänge bei Wirbellosen und Wirbeltieren ein Vergleich zulässig ist (Ausschluss des Fehlers 2).

Unsere planmässig angestellten Versuche sollen Art und Umfang der Nervenwirkung auf die Regeneration zeigen. Wir gehen daher von der unverletzten Innervation aus und untersuchen das Verhalten des Nervensystems während des Regenerationsprozesses. Dadurch werden die Fehler 4, 5, 6 und 7 vermieden.

Die Experimente wurden an 125 bis 150 mm grossen Axolotln (*Siredon pisciforme* Shaw) vorgenommen, die unter gleichen Bedingungen bei einer Temperatur von 18—20° C gehalten wurden. Untersucht wurde die Regeneration der linken Hinterextremität nach Amputation in der Mitte des Oberschenkels.

Die Innervation geschieht durch drei Spinalnerven des Lumbosacralmarks (das 15. 16. 17. Paar des Rückenmarks), die zusammen den Plexus lumbosacralis bilden. Der Nervus cruralis und der Nervus ischiadicus mit ihren Verzweigungen versorgen die Extremität. Das morphologische Bild (Abb. 1.) zeigt in Bezug auf die Anastomosen topographische Variationen. Dazu kommen zuweilen noch morphologische Variationen derart, dass ein vierter dünner Spinalnerv oral- oder caudalwärts von den genannten drei an der Plexusbildung teilnimmt.

Unser Eingriff bestand in der völligen Resektion des Plexus nach Durchschneidung der Spinalnerven oberhalb des Plexus. Es wurde also darauf geachtet, dass nicht einzelne der drei Spinalnerven oder ein accessorischer erhalten blieben.

1. Kontroll-Reihe für die Regeneration nach Amputation ohne Resektion. Knospe durchschnittlich nach 12, Anlage mit innerer Differenzierung nach 26, äussere Differenzierung nach 35 Tagen. Atypien der Form (Schwimmhäute zwischen den Zehen, Verwachsung einzelner Zehen). Volle Funktion des Regenerats, besonders Bewegung des Knie- und Fussgelenkes. Mit der funktionellen Ausgestaltung wird die Grenze zwischen Amputationsstumpf und Regenerat verwischt.

2. Kontroll-Reihe für die Nerven-Regeneration. Resektion des Plexus lumbosacralis ohne Amputation. Das Bein ist gelähmt und reagiert nicht auf Berührungsreize. Nach 15 bis 28 Tagen kommt eine neue Innervation zustande. Zuerst tritt Reaktion auf Berührungsreize, dann aktive Bewegung auf. Die Bewegung beginnt proximal im Hüftgelenk. Das Knie bleibt sehr häufig steif. Schliesslich wird die Bewegung der Fussgelenke hergestellt. Die Bewegungen des linken, operierten Beines sind mit dem rechten, nicht operierten koordiniert. Die dauernden Funktionsatypien bestehen ausser in der häufigen Versteifung des Kniegelenkes darin, dass der Fuss nicht mit der Sohle, sondern nur mit dem Fussrand aufgesetzt

wird. Die Obduktion zeigt in verschiedenem Masse atypische Bilder der Innervation (Abb. 2). Nicht immer treten drei Spinalnerven zur Plexusbildung zusammen, sondern zuweilen nur zwei. Ein Nerv hat nicht regeneriert. Im Plexus finden sich Autoanastomosen. Die Innervation des Beines ist unregelmässig. Die Nerven sind zuweilen auffallend dünn. Dem Funktionsausfall entsprechen also Mängel in der durch Regeneration hergestellten Innervation. Wiederholte Resektion ergibt dasselbe.

3. Gleichzeitige Amputation und Resektion. An dem im Hüftgelenk gelähmten Stumpf kommt die Regeneration in Gang, erreicht nach 11 bis 16 Tagen das Knospenstadium und bleibt dann im Vergleich zur Kontrolle 1 zurück: Anlage mit innerer Differenzierung nach 38 bis 54 Tagen und weiter entsprechende Verzögerungen. Formal zeigen die Regenerate die gewöhnlichen Atypien. Sie sind kleiner als nach Amputation ohne

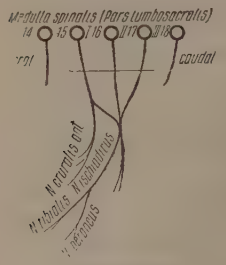

Fig. 1.

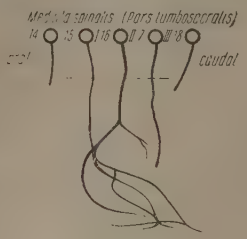

Fig. 2.

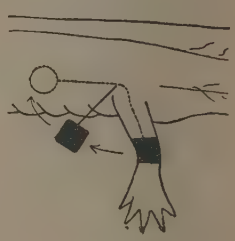
Fig. 3.

Resektion. Die Nervenregeneration erfolgt wie in Versuchsreihe 2 mit demselben Ergebnis. Innervations- und Funktionsatypien, leichte Bewegungsbehinderungen, denen Innervationsmängel entsprechen.

4. Amputation mit folgender Resektion auf verschiedenen Regenerationsstadien. a) Resektion im Stadium der Knospe 13 Tage nach der Amputation. Die Regeneration des gelähmten Beines geht weiter. Die innere Differenzierung der Anlage erfolgt nach 26, die äussere nach 32 bis 36 Tagen. Zeitliche Verzögerung findet also nicht statt. Das Regenerat gleicht auch formal der Kontrolle; nur ist es erheblich kleiner (vor der Funktionsaufnahme durchschnittlich 4 mm gegen 12 mm der Kontrolle). Bei der Funktionsaufnahme zeigen sich dieselben Funktionsatypien wie in 2: Knieversteifungen, Kanten des Fusses. Das funktionierende Regenerat wächst nach. Die Regeneration wird also nach der Enervation fortgesetzt und die Innervation folgt als weiterer regenerativer Prozess.

b) Resektion im Stadium der beginnenden inneren Differenzierung der Anlage 25 Tage nach der Amputation. Die innere und äussere Differenzierung wird durch die Enervation nicht verlangsamt (äussere Differenzierung 34 Tage nach der Amputation). Die Regenerate sind kleiner als in der Kontrolle. Nach der verzögerten Funktionsaufnahme zeigen sich Funktionsatypien: Knieversteifungen, Kanten des Fusses, ruckartige Bewegungen, die nicht mehr ausgeglichen werden.

c) Resektion im Stadium der äusseren Differenzierung der Anlage 38 Tage nach der Amputation. Auf die Ausdifferenzierung der Anlage hat die Resektion keinen Einfluss. Dagegen wird das Wachstum sehr verlangsamt. Erst drei Monate nach der Amputation beginnen die kleinen Regenerate zu funktionieren und nachzuwachsen. Die Funktionsatypien (Knieversteifung, erhebliche Behinderungen im Fussgelenk, Aufsetzen des Fusses mit dem Rand) werden nicht mehr ausgeglichen.

5. Deviation des N. ischiadicus mit Transplantation eines Beinstückes, das das Kniegelenk enthält (Abb. 3). Die Verlagerung wurde oral- oder dorsalwärts vorgenommen. Der Stumpf wurde exartikuliert. Bei gutem Verschluss der Exartikulationswunde durch Hautimplantate erfolgt an dieser Stelle nichts. Nach Einheilung beginnt die Regeneration an der Wundfläche des verlagerten Unterschenkels. Es wird ein Fuss mit Fussgelenk gebildet. Nach der Ausdifferenzierung tritt Funktion ein, die mit dem nicht operierten rechten Bein koordiniert ist. Das Kniegelenk bleibt durch Verwachsungen versteift. Die Regenerationszeit variiert sehr (äussere Differenzierung frühestens nach 35, spätestens nach 65 Tagen), was ohneweiteres auf die Schwere des Eingriffs zurückzuführen ist. Die Regeneration des Transplantats geschieht herkunftsgemäss. Die Funktion entspricht der Form. Die Innervation war nie unterbrochen.

Fig. 4.

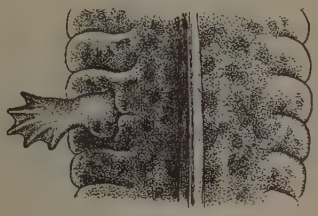

Fig. 5.

6. Deviation des N. ischiadicus ohne Transplantation eines Beinstückes. Nach künstlichem Verschluss der Exartikulationswunde wird der N. ischiadicus oval- oder dorsalwärts verlagert (wie in 5). Irgendwelche Regenerationsvorgänge in der Gegend des Nervenendes finden nicht statt.

7. Transplantation eines Beinstückes ohne Deviation des Nerven in die Extremitätregion. Die Exartikulationswunde wird mit Haut verschlossen. Verpflanzt wird das das Kniegelenk enthaltende Beinstück. Die Operation ist also dieselbe wie bei der Deviation (5) mit dem Unterschied, dass der N. ischiadicus nicht mit verpflanzt wird, das Transplantat also ohne Verbindung mit dem zentralen Nervensystem ist. Vom Transplantat aus erfolgt Regeneration. Die Verzögerung im Auftreten der Stadien (Knospe nach 19—28, innere Differenzierung der Anlage nach 53—70 Tagen) erklärt sich daraus, dass das Transplantat erst einheilen muss, ehe es ernährt werden kann. Regenerate bleiben klein und funktionslos (Abb. 4). Sie sind nicht innerviert.

8. Transplantation eines nervenhaltigen Oberschenkelstückes in den Rücken. Das Transplantat stammt aus einem intakten Bein. An der Amputationsstelle erfolgt Regeneration (wie in der Kontrolle 1). Das Transplantat liefert ein funktionsloses Regenerat, dessen Stadien verzögert auftreten (Knospe nach 18 Tagen, Anlage nach 42 bis 60 Tagen). Der Regeneration muss die Einheilung des Transplantats vorangehen. Die Regenerate sind nervenlos. Nach abermaliger Amputation regenerieren sie aufs Neue.

9. Transplantation eines nervenlosen Oberschenkelstückes auf den Rücken. Dieselbe Operation wie in 8 mit dem Unterschied, dass die Transplantaten nicht innervierten, funktionslosen Extremitäten von Tieren der Serie 2 entstammen, also noch

nicht innerviert waren und deshalb auch keine Nervenreste enthielten. Die Regeneration gleicht der in 8 mit derselben Stadienverzögerung und denselben funktionslosen Endgebilden (Abb. 5).

Ergebnisse. Die Regeneration kommt am Amputationsstumpf der Hinterextremität in Gang und führt zu einem Endgebilde, gleichgültig ob der Plexus lumbosacralis vorhanden ist oder nicht. Auf die spezifisch-morphogenetische Determination der Regeneration hat die Verbindung des Regenerats mit dem cerebro-spinalen System keine Wirkung. Das cerebro-spinale System gehört nicht zu den Determinationsfaktoren. Die ohne Verbindung mit dem Nervensystem gebildeten Regenerate werden als funktionslose Endgebilde stationär. Ihre Atypie ist auf diesem Stadium dieselbe, wie wenn der Amputationsstumpf innerviert wäre (Vergleich der Serien 1 und 3). Die Innervation ist selbst ein regenerativer Prozess (Serie 2) und findet überall da statt, wo Nerven vom Zentrum in das Regenerat einwachsen können (Serien 3, 4, 5). Die Regeneration geht nur von einem Amputationsstumpf, nicht vom Nerven allein aus (Serien 6, 7). Die An- oder Abwesenheit von Nervengewebe ohne Verbindung mit dem Zentrum macht keinen Unterschied (Serien 8, 9). Die regenerative Innervation folgt der Anlagendifferenzierung, besonders der Bildung von Muskeln. Im Regenerat kommen mit seiner Innervation alle die Bildungen in Gang, die zur funktionalen Ausgestaltung des Endgebildes gehören. Das cerebro-spinale System wirkt über die Funktion der Organgewebe auf die Ausgestaltung des Regenerats. Es wirkt also nicht als spezifisch-morphogenetischer Determinationsfaktor, sondern als ein stimulierend-trophischer Realisationsfaktor.

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Eingegangen
d. 5. V. 1939.

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Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1939. Volume XXIII, № 9

DEVELOPMENT MECHANICS

HETEROGENEOUS INDUCTION OF THE EXTREMITY IN *TRITON VITTATUS*

By A. I. MACHABELI

(Communicated by I. I. Schmalhausen, Member of the Academy, 5. V. 1939)

The phenomenon of heterogeneous induction of an extremity first discovered by B. I. Balinsky in *Tr. taeniatus* in the course of his experiments with the transplantation of the auditive vesicle ([1]) was later confirmed by himself and by other investigators both in this and in other species of amphibiae ([2,3,4,5] et al).

The results of these studies showed that the development of an additional extremity under the influence of the auditive vesicle or of other inductors (olfactory placode, rudimentary hypophysis) is a property of the side region of embryos and early larvae of both caudate and tailless amphibiae. However, the ability to develop additional extremities proved to be different in different representatives of the amphibiae. The highest ratio of inductions was obtained in *Amblystoma punctatum* (48%). An approaching, though somewhat lower percentage was observed in *Tr. taeniatus* (39%); the next place is occupied by *Tr. cristatus* and *Amblystoma mexicanum* ([7]).

As this is visible from the studies by Filatov and Balinsky ([4,5,9]), tailless amphibiae show a much weaker ability, compared to caudate ones, to form additional extremities. The formations arising as the result of induction do usually not reach the stage of development of the extremity which is observed in the caudate amphibiae.

Comparative experimental investigation of the process of heterogeneous induction of an extremity and of other organogenies permits, by revealing their peculiar characteristics in different species and by augmenting the number of facts, to inquire deeper into the phenomena under study.

The importance of this trend in experimental embryology led by Filatov, is confirmed by a number of researches concerning the development mechanics of various organogenies. Suffice to cite the works on lens-formation in amphibiae, as well as the experiments of Filatov with the transplantation of the auditive vesicle in amphibiae and fishes. The discovery of the phenomenon of heterogeneous induction of the extremity is also due to such comparative studies.

The purpose of the present work was to study the question of the heterogeneous induction of an extremity in the Asia Minor triton (*Triton vittatus*) which has not been tested earlier in this respect.

Embryos of *Tr. vittatus* developing from spawn laid in the aquarium were used as the material for these experiments. The auditive vesicle served as the

inductor. The transplantation was homoplastic. The auditive vesicle, cleaned thoroughly from the surrounding mesenchyma cells, was planted into an epithelial pocket made in the side of the recipient in the region of the 7—8th somites. 1—1.5 hours later the operated embryos were subjected to an examination for verifying whether the graft has been retained or not.

The recipients were embryos at 3 stages. The embryos of the earliest of these stages are somewhat curved, their tail being bent under. The branchial tuber is continuous. Supporting filaments in the shape of small tubera. Such embryos correspond to stage 33 of Harrison's table.

The embryos of the next stage are straightened, have a longer tail which is less bent under; it forms a right angle with the longitudinal axis of the body. The branchial tuber is divided into lobules by shallow furrows. The supporting filaments have the shape of somewhat elongated tubera (stage 34).

The recipients of the eldest of the stages taken for the experiments are completely straightened embryos with a straight or but slightly bent tail, but without a natatory trimming. The supporting filaments have already the shape of more elongated tubera, or rather of short processi (stage 36).

As donors in every case were used only embryos of this last stage (No. 36).

The operated embryos were grown in crystallizers. Food: daphniae and cyclopes. 20—40—60 days after the operation the larvae were fixed in a sublimate solution with acetic acid. They were stained totally with boric carmine and the sections were stained supplementarily according to Mallory or with Bismark-braun.

Experimental Results. A total number of 56 larvae were fixed. In three cases the induction of a developed free extremity was obtained and in 23 cases cartilages of the character of an extremity had formed. Since all the three induced free extremities were identical in their structure and their development was similar, I will describe only one of these cases.

10. VI. 1938. An auditive vesicle is transplanted into the side of an embryo at stage 36. The donor is of the same stage. On June 20, i. e. 10 days after the operation, an outgrowth possessing the shape of a small spade appeared at the site of the graft in the middle between the anterior extremity and the place of the posterior extremity. Later on this outgrowth developed into an additional extremity. By this time the normal anterior extremities had already 3 fingers each, while the posterior extremities were as yet absent. The larva was fixed on August 5, when the posterior extremities had already 3 fingers each. The heterogeneously induced extremity was found to be straight, harmonious to the anterior one, possessed 4 well developed fingers, a metacarpus, radial and ulnar bones, a well pronounced elbow joint and a forearm. On the 1st and 4th fingers, somewhat above their bases, a rudiment was formed on each; two similar rudiments which had developed from the metacarpus, were present at the interior surface of the extremity.

The cartilages, bearing the character of an extremity, consist of young cells, histologically identical with those of the cartilages of normal extremities. The induced cartilages have an elongated oval, egg-like or rod-like shape, and almost in every case are surrounded by a small number of mesenchyma cells. In some preparations—those of larvae fixed at earlier dates—it is particularly well visible that the process of cartilaginization begins from within the mesenchymatous accumulation. Sometimes cartilages are encountered which have a considerable thickening at one of the ends, reminding one of the articulation ends of the humerus or of the femur. The orientation of such cartilages varies: their proximal part is in some cases directed towards the auditive vesicle, in others—towards the side of the larva.

The topographic disposition of the cartilages described is also characteristic. All of them without exception have formed between the auditive vesicle and the epithelium and in no case have cartilages of this shape developed between the auditive vesicle and the somite, i. e. the pleura. This circumstance, regarded in the light of Filatov's conceptions ([3],[4]) on the formation of the initial stage in the development of the extremities, along with the histological characteristics of the cartilages, confirms their extremity character.

It is interesting to note that cartilages of this kind either lie at some distance from the surface of the graft or touch it only partly.

The results of the experiments were different for the 3 stages tested. Out of the larvae operated at the earliest stage tested (stage 33) 34 were fixed (the larvae were fixed 20 or more days after the operation). Induction of a free extremity was obtained in none. In 16 cases as the result of induction cartilages were formed bearing the character of an extremity which I have described above.

Operated embryos of the next stage (stage 34) were fixed in the number of 12. In two of them the induction of an additional extremity, similar to the one described above, has taken place. In 7 larvae cartilages of an extremity type have formed. Out of the larvae operated at the eldest stage (stage 36) 10 were fixed. In one case developed the additional extremity described above. In the remaining 9 cases there was no cartilage formation of any kind under the influence of the transplanted auditive vesicle. In none of these cases could we detect any traces of mesenchyma accumulation.

Alongside of the formation of cartilages of the extremity type we observed almost in every case the formation of capsular cartilage. The cells of such cartilage are older and larger than those of the extremity cartilages; they stain less intensely with carmine. But in none of the cases did these capsular cartilages give rise to a complete capsule; they represent in most cases narrow strips, closely clinging to the surface of the labyrinth for a relatively small space, never exceeding one half of its circumference.

Trunk mesenchyma should be assumed to be the material from which these capsular cartilages are formed. In behalf of this speaks, on one hand, the circumstance that the transplanted auditive vesicle had been preliminarily thoroughly cleaned from the mesenchyma surrounding it (this was easy to achieve at the stage at wich the donor was at the moment of transplantation) and, accordingly, no head mesenchyma was grafted together with the vesicle.

On the other hand, the origin of these capsular cartilages from trunk mesenchyma is corroborated by the following considerations. The age of such cartilage was in none of the cases elder than that of the head cartilage of the same larva, but usually somewhat younger than it, while the auditive vesicle in most cases (i. e. when the recipients belonged to one of the two first stages taken for these experiments) was obtained from elder embryos, and in the case if head mesenchyma had been carried in together with it, a cartilage of an elder age would have developed than the one actually observed in the sections. The relatively small size of these capsular cartilages cannot be used as an argument for the assumption that they have arisen from head mesenchyma brought in together with the graft, since even if it were brought in, its amount could not have been sufficient for the formation of cartilages of the size observed.

As may be seen from the above, these experiments have shown that the lateral region of *Tr. vittatus* is able to develop, under the influence of the transplanted auditive vesicle, an additional free extremity, similarly to the other earlier investigated species of tritons (*Tr. taeniatus*, *Tr. cristatus*).

Besides, the development of this additional extremity, the rate and dates of its development as well as its character, are similar to those observed by Balinsky in *Tr. taeniatus*. Yet, alongside of this qualitative similarity, there exists between these species of tritons a noticeable distinction of a quantitative character, in the sence of the frequency of induction of a free extremity. In this respect *Tr. vittatus* occupies an intermediate position between *Tr. taeniatus* and *Tr. cristatus*.

According to the data of Balinsky ([2],[3],[7]) induction of an additional free extremity in *Tr. taeniatus* under conditions similar to those of our experimets (inductor—auditive vesicle; the same site of transplantation)—occurred in 6 cases out of 21, in 4 out of 33 and in 3 out of 39. In the experiments of Popov ([8]) an additional free extremity was formed under the influence of the auditive vesicle in 8 cases out of 52. The frequency of induction in *Tr. cristatus* is much lower than in *Tr. taeniatus*: an additional free extremity was formed only in 1 case out of 75, and even this one was a malformation.

In *Tr. vittatus* an additional extremity was formed in 3 larvae out of 56. This points to that the lateral region of *Tr. vittatus* possesses a higher ability for the production of an additional extremity than that of *Tr. cristatus*, but a considerably lower one compared to *Tr. taeniatus*. If we take into account, moreover, the circumstance that owing to hot weather a part of the operated larvae died 1—1.5 months after the operation without any outward sign of a growing additional extremity which ought to have already developed by that date, the ability of *Tr. vittatus* to form induced extremities should be considered to be actually still lower than in the case if only the number of those larvae which had been fixed is taken into account.

Along with the relatively low frequency of the induction of a free extremity, it is necessary to note the considerable number of cases in which cartilages of the extremity type (23 cases out of 56) were formed; this exceeds the frequency of the formation of similar cartilages in *Tr. taeniatus*, as registered by Balinksy. This circumstance, as well as the absence of any noticeable accumulation of mesenchyma around the graft and the forming cartilages, gives us ground to assume that the cause of the relatively low frequency of the induction of free additional extremities is a mesenchyma deficiency within the sphere of action of the inductor. The experiments described fail to give any answer as to the eventual causes of this deficiency.

If we consider the results obtained in each of the embryonal stages tested, it seems interesting to note that in the larvae operated at stage 33, a free additional extremity has been formed in none of the cases, while out of 12 larvae operated at stage 34 such an extremity was induced in 2 cases, and out of 10 larvae operated at stage 36—in 1 case, while in the remaining 9 even no cartilages were formed and no traces of mesenchyma accumulation could be observed in the vicinity of the graft. In view of the scarcity of material of the two last stages and of lack of material of still older ones, it is impossible to draw a definite conclusion regarding the peculiarities of induction at the different stages, and one can but endeavour the supposition that *Tr. vittatus* possesses an ability to the formation of induced extremities during a shorter period than *Tr. taeniatus* ([6],[10]).

The results of the described experiments permit to draw the following conclusions:

1. The lateral region of *Tr. vittatus* is able to develop under the influence of a transplanted auditive vesicle an additional extremity.

2. The frequency of this induction of a free additional extremity in *Tr. vittatus* is considerably lower than in *Tr. taeniatus*, and higher than in *Tr. cristatus*.

3. The trunk mesenchyma of *Tr. vittatus* may transform into cartilage of a capsular type under the influence of the auditive vesicle.

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of the People's Commissariat
for Education of the Georgian SSR.

Received
5. V. 1939.

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Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1939. Volume XXIII, № 9

DEVELOPMENT MECHANICS

EXHAUSTION AND RESTORATION OF THE INDUCTIVE POWER OF THE EYE CUP IN REPEATED INDUCTIONS IN *ANURA*

By N. A. MANUILOVA

(Communicated by I. I. Schmalhausen, Member of the Academy, 5. V. 1939)

The discovery of inductive influences of some parts of the developing organism upon the others, influences which constitute the necessary conditions of development and are present at any moment, almost immediately called forth the study on the nature of this process. However, we still are at loss to know what are the processes that occur during induction and we judge of them only by their manifestation in morphogenesis. Two ways of inquiry may be distinguished here: 1) the attempt to reduce the formative phenomena to physico-chemical processes and 2) the study of the principles of the inductive influences in their morphological expressions. Unfortunately, one often has to meet with a strong tendency of mutual desapproval of the two trends. This is especially true of the adepts of the former. Suffice to refer to the text-book by Huxley and de-Baehr (1) where it is stated point-blank that the experiments with non-living organizers are expected to bring the solution of chemical process of determination, and that experiments in this direction are of greater interest than any other modern work in development mechanics.

There is no doubt, however, that the two trends should complement each other and their amalgamation has to yield the most fruitful method for studying the induction process.

It was my intention to demonstrate by experiments with repeated inductions what value in studying the nature of the induction process should be ascribed to such experiments which take into account only the morphological manifestation of the inductive influence.

The present work is a continuation of the experiments conducted in 1937 and already published (2). I have succeeded in demonstrating by means of repeated removals of the anlage of the lens that the lens-forming power of the cup is exhausted in case of frequently repeated induction. On the other hand, it was noted in these experiments that occasional increase in the time interval between two actions of the cup upon the epithelium had a favourable effect upon its lens-forming capacity. This fact suggested further studies; I have made an assumption as to the possibility of restoring the lens-forming power of the cup when operations are performed at longer intervals. An attempt is made in the present paper to prove this. It was necessary, as in the preceding study, to make one and the same cup repeatedly act upon the

epithelium at different intervals between the operations. In case the cup was able to restore its lens-forming capacity, the effect of its action should be the better, the longer the interval between inductions.

The work has been done in the Borjomi region on embryos and tadpoles of *Peloclites caucasicus*. Not more than 4 operations were done on one and the same animal, neither was each animal operated four times. As for the sake of comparison it was important to know the effect of the cup upon the epithelium not merely after the 4th but after the 1st, 2nd and 3rd operations, too, a part of embryos had to be fixed after every operation and before the one to follow it; the investigation was done on sectioned material.

The lens epithelium was removed at the first operation in such a way that the cup came into contact with the epithelium which covered the wound after the operation. In all the three following operations I have removed only the anlage of the lens (in case it was formed) and transplanted instead of it a bit of the body epithelium taken always at the stage of early tail bud. Three basic experimental series were arranged; they differed in the intervals between operations, the interval being two days in the first series, 6 days in the second and 10 days in the third one.

A histological study of the material has shown the length of the intervals between operations to have essential significance for the lens-forming capacity of the cup. The longer the interval between operations, i. e. between the moments when the cup had to act again upon the embryonic epithelium, the oftener lenses were formed and the better they developed even after the 3rd and 4th operations. At two-day intervals the lens-forming capacity of the cup very sharply drops after several inductions: after the 4th operation when the cup acts the 5th time upon the epithelium (the first induction occurs in normal development) only in one tadpole out of the 12 successfully operated ones a small hollow lens was observed, whereas in the remainder there were observed no signs of transformation of the epithelium into a lens.

At six-day intervals the picture of lens formation is changed. After the 2nd and 3rd operations formation of lenses is observed more frequently than in the first series, although not in all the embryos. Besides, the lenses formed do not show any signs of defective development and are differentiated, as usual, into epithelium and cells of fibres. After the 4th operation in 7 embryos out of 19 well differentiated lenses were formed, while at two-day intervals only in one of the embryos the transplantate was observed to transform into a lentoid. Finally, at 10-day intervals nearly in all the tadpoles operated good lenses developed after the third operation. It was noted even after the 4th operation that the lenses on the operated side were but slightly smaller than on the non-operated one. It is possible, however, that later on this difference becomes levelled. One could argue that the lenses were better developed in the 10-day series than in others, owing to the fact that they were formed from the edge of the cup. However, in tailless amphibiae the lens fails to form as easily and as often as it was observed here; besides, a 10-day interval is not sufficient for a full separation of such a lens from the edge of the cup. All the lenses formed were located in the pupil-perforation and were surrounded by a greater or smaller quantity of pigment cells which remained from the transplantate.

The above data enable one to draw a definite conclusion regarding the lens-forming capacity of the cup. First, on the basis of the 3rd experimental series, it can be said that the lens-forming capacity of the cup remains almost unchanged by several inductions. Such a conclusion seems to be at variance with that made from the previous work. However, this contradiction is a deceptive one. Both conclusions are correct. Every induction is followed

by an expenditure of the formative capacity, yet, as shown by the experiments of the 3rd series, this capacity may be restored under suitable conditions.

And not only is correct the conclusion previously arrived at to the effect of the exhaustion of the morphogenetic properties of the cup, but any inference deduced from the 3rd series of the given work as to the stability of the lens-forming capacity of the cup in case of inductions would be erroneous without these experiments. Only a successive increase of the intervals between operations enabled me to notice that the induction was connected with an expenditure of the morphogenetic capacity of the cup; however, this capacity could be restored, perhaps even attaining its original level.

These experiments in which only the morphological effect was taken into consideration thus show that induction is connected with expenditure of formative energy which seems to be the result of a more active physiological condition of the cup at this moment. Therefore, if the data on the disappearance of glycogene in the cells of the organizer ([3],[4]) point to the possibility of energy consumption at the time of induction, the purely morphological experiments reported here make it possible to assume not only the consumption of energy, but its further restoration as well.

The following conclusions may be arrived at on the basis of the data reported. 1) Each lens-induction is connected with the consumption of the formative energy of the cup. 2) The cup is able to restore the energy consumed. 3) A definite minimum time is required for restoring this energy; in the reported case it equals approximately 10 days.

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Received
5. V. 1939.

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Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1939. Volume XXIII, № 9

DEVELOPMENT MECHANICS

STUDIES ON LENS-FORMING PROPERTIES OF BODY EPITHELIUM IN *TRITON VITTATUS*

By T. A. SIKHARULIDZE

(Communicated by I. I. Schmalhausen, Member of the Academy, 5. V. 1939)

Experiments on early removal of eye primordia which have been first done by Spemann (1) and Lewis (2) have shown that in the majority of amphibia no lens is formed in absence of eye. On the basis of these data the two authors arrived at a correct conclusion on the formative rôle of the cup. However, the cup was tried in these experiments against the lens epithelium which in certain amphibia is in such a high measure ready to autonomous development that a lens-formation may be started even in absence of the eye. In order to find out what is the lens-forming capacity of the cup (irrespective of the readiness of the epithelium to autonomous development) one should try it against a neutral epithelium, i. e. such one which does not participate in lens-formation during typical development. Such experiments conducted on the body and head epithelium have shown these epithelia to develop into a normal lens under the influence of the cup. Hence it followed: 1) that the eye cup is capable of autonomous setting and development of a normal lens and 2) that a normal lens can develop from body and head epithelium, provided they are brought into contact with the eye vesicle. Transplantations of the latter under the body epithelium, or vice versa, of the body epithelium on the eye have demonstrated a different readiness of the epithelium for lens-formation in different species. Spemann from his 1912 work (3) arrived at a further conclusion that there are sharp interspecific differences in respect to lens-forming properties of the body epithelium. He has shown that, while in *Rana fusca* the body epithelium was easily transformed into a lens under the influence of the transplanted eye, not a single case of lens setting could be observed in Rana *esculenta*.

During the recent years the number of similar experiments on the properties of the body epithelium in different species considerably increased (4,5,6,7); the results of these experiments have shaken Spemann's attitude toward the existence of a strict specifity of the properties of the body epithelium. It proved possible to establish a series of gradual transitions from species in which the lens is easily developed from the body epithelium to such ones in which lens formation occurs with considerable difficulty.

The properties of the body epithelium of the amphibia from waters of Moscow region have been studied rather fully. The results of such studies made it possible to start experiments on a still larger scale and to study the properties of the body epithelium in these species at different stages of development. Experiments arranged in such a way have shown that in the

amphibia studied the ectoderm of the gastrula develops equally well into a lens, and that subsequent differences in lens-formation which have been established in these species at the stage of the tail bud probably depend merely on unequal rate of epithelia differentiation.

Owing to the fact that *Triton vittatus* which is the object of experimental studies in Transcaucasia has not been at all investigated in this respect, I have arranged a series of experiments on the properties of the epithelium in this species. The results of the first experimental series are communicated here; they bear on the body epithelium alone.

Embryos of *Triton vittatus* from the Tbilissi waters served as experimental material.

In order to investigate the lens-forming properties of the body epithelium eye primordium was transplanted into the side of the embryo. Embryos at two stages served as recipients. At the first of them (29 following the table of Harrison) the embryo is bent, branchial tuber is continuous, no changes are observed in sections of the prospective lens epithelium. At the second stage (34) the embryo is considerably elongated, branchial tuber is already subdivided into lobes. The lens primordium when studied on sections has the appearance of polystychious thickening of the epithelium.

Embryos at the same two stages served as donors. Aside from this in some series the eye vesicle was taken at the stage of early tail bud.

Altogether 84 embryos were operated; fixed and examined on sections were only 46, since the remainder were operated during the hot period and died soon after the operation. The embryos were fixed 2—3 days after the vesicles were transplanted.

The experiments have shown that the eye primordia in *Triton vittatus* very rarely develop into a cup when transplanted. In the first series, the eye vesicle being transplanted at the tail bud stage, regular cups have developed only in 5 embryos out of 20. In the remaining upon examination on sections the transplantate proved to consist of regions of retina with a pigment membrane. In the second series where embryos at later stages (29 or 34) served as donors, the transplantate not only failed to transform into a cup but, in several embryos it was even broken up into separate cells.

The formation of the lens was observed only in those five embryos in which regular cups have developed. In three of them the lenses had already separated from the epithelium so that one could hardly be sure that they were derived from epithelium; it is true, there was no evidence of their having originated from eye material, either. In two cases lenses had an appearance of vesicles which still were connected with the epithelium. This means that these lenses have actually developed from the body epithelium.

On the basis of the above data we are justified in the conclusion that the body epithelium in *Triton vittatus*, like in other amphibiae, is capable of development into a lens under the influence of the eye. As to the readiness of its response to the action of the cup, no answer can yet be obtained from the data available, since in the majority of the embryos the transplantate failed to transform into a cup, and the lens was formed only when a regular cup had developed. It is possible that formation of the lens from the body epithelium would be observed more often had the transplanted eye developed regularly.

Conclusions.

1. The body epithelium in *Triton vittatus* develops into a normal lens when it comes into contact with a transplantate which has developed into a regular cup.

2. The eye vesicle in *Triton vittatus* rarely develops into a regular cup when transplanted under the body epithelium.

3. When the eye primordium is transplanted under the body epithelium at the stages 29 or 34, it fails to develop into a regular cup and, moreover, in some cases breaks down into separate cells.

Embryological Station of the
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Received
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Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1939. Volume XXIII, № 9

DEVELOPMENT MECHANICS

SEQUENCE IN THE ONTOGENETIC APPEARANCE OF VERTEBRAL ARCHES IN TELEOSTS AND THE OMISSION OF CHONDRAL STAGES IN THEIR DEVELOPMENT

By S. V. EMELIANOV

(Communicated by I. I. Schmalhausen, Member of the Academy, 9. V. 1939)

In my works on the morphology of ribs ([1],[2]) I was discussing the question of the omission of cartilaginous stages in the development of chondral ossifications. Proceeding with the study of the phenomenon of the omission of cartilaginous stages in the ontogeny, I am giving in this paper a brief description of the results of my observations on the development of vertebral arches in bony fishes.

The arches appear in different segments of the same fish at different time. Moreover, the arches in different segments of the same fish may exhibit a variety in the mode of their development: in some segments the arches do pass through cartilaginous stages, in others they do not.

The object of my present work was first to determine the sequence in the appearance of arches in different segments (this sequence has not been studied up to the present), and secondly, to make clear whether the same regularity is observable in the development of arches as in that of ribs, i. e. whether cartilaginous arches in the same form arise before the bony ones.

The study of a number of bony fishes has shown that, generally, the sequence in the appearance of arches is in all forms alike. There are, naturally, some individual variations, but they do not change the general character.

The teleosts have two centra from which the development of arches begins —the occipital arch and the first hypural. The occipital arch (the neural arch) is the first to appear. Somewhat later appears in the tail the most cranial hypural. From these two points proceeds the further development of the arches. In the tail soon after the appearance of the first hypural are formed in craniocaudal direction the following hypurals, and in caudocranial direction are gradually appearing the haemal arches. More or less simultaneously with the appearance of the ventral elements in the distal caudal region are forming the neural arches of the anterior trunk region. The formation of the upper arches begins from the first vertebra and extends backwards. Afterwards, above the last caudal haemal arch appears the neural arch, and forwards from this arch in the caudocranial direction develop arches of the anterior segments. Somewhat later appear the haemal arches in the anterior trunk region. Then, gradually, arise the haemal arches of the posterior trunk segments. Lastly are produced (appearing in craniocaudal

succession) the arches over the hypurals. There are very few of these neural arches in teleosts; they are rudimentary and of elongated (craniocaudal) form. There is a considerably greater number of such arches developed in *Amia*.

The vertebral arches appear in teleosts within a very short time so that it is not always an easy matter to watch the succession of their appearance. A large number of young fish of about the same age is required. In the present brief article I am unable to dwell on the question as to what this quite intricate succession in the appearance of arches means, and to what extent the ontogenetic sequence recapitulates the phylogenetic one. I shall note only that the caudocranial successive appearance in the ontogeny of some characters, like, for instance, the fusion of the arches of the right and left sides, the development of neural and haemal spines, coincides with their appearance in the phylogeny, as indicated in my earlier work (3).

Among the bony fishes investigated by me, only in *Salmo* the arches in all segments pass through chondral stages (*).

In *Amiurus* only those of the modified upper arches do not pass through the chondral stages which are arranged above the hypurals. They are the last to appear (in young specimens more than 10 mm in length).

In *Perca* the chondral stages are absent in the same upper arches as in *Amiurus*, as well as in all haemal arches of the trunk.

In the 10.5 mm perch all chondral elements are developed, but no bony arches are present. In the 11.5 mm young specimen the bony haemal arches have appeared but in the two terminal trunk vertebrae; the bony arches adjoin caudally the cartilaginous haemal arches which have previously appeared. In the 12.5 mm specimen bony haemal arches are present in the whole trunk. In the region of hypurals the bony neural arches have developed. The bone in those neural arches is somewhat younger than the bone in the haemal arches of the trunk.

In *Rutilus rutilus* chondral stages are passed by the first 6 neural arches of the trunk, nearly 20 anterior haemal arches, 5 terminal caudal haemal arches, 5 terminal neural arches and all the hypurals. No chondral stages are passed by the neural and haemal arches of the middle of body, and the neural arches over the hypurals. In the 8.5 mm *Rutilus* the chondral elements alone are present. In the first five trunk vertebrae the neural and haemal arches are developed. The extent of development of the cartilage in these arches is not the same, in the more proximal arches the cartilage is better developed. In the tail are developed 5 cartilaginous hypurals with two haemal arches in front of them—one cartilaginous and one prochondral. Above these heamal arches two prochondral neuras arches are developed. The remaining body segments have neither chondral nor bony arches. In the 9 mm *Rutilus* an appreciably higher degree of development has been attained; the cartilage is developed in 6 anterior neural arches, in about 10 haemal arches, in the hypurals and in 5 haemal and neural arches lying in front of the hypurals. At the same time the bony arches are beginning to appear—the bone being developed in a considerable part of the neural trunk arches, beginning from the seventh arch. At the end of the trunk and the beginning of the tail there are neither cartilaginous nor bony neural arches present, though above the hypurals bony upper arches have appeared. In the 10 mm young specimen all neural arches are developed. All the cartilaginous haemal trunk arches are developed (the posterior ones are prochondral). The bony

* In the present paper I am not dealing with a case of omission of cartilage in a part of the arch. Chondral arches are called here those having at least a small cartilaginous part.

haemal arches have not yet made their appearance. In the 12 mm young specimen all the neural and haemal arches are developed.

In *Gobius* but the hypurals, 4 most caudal haemal arches and 4 neural arches situated above these chondral haemal arches, pass through cartilaginous stages. Arches of the rest of body segments, particularly those of all trunk segments, do not pass through cartilaginous stages. The material at my disposal on the development of *Gobius* was inadequate, and therefore I was unable to determine the complete sequence in the development of the arches. All I can say is that the 6.25 mm young specimen exhibits the beginning of the chondrification process in the hypurals with no traces of bone in other vertebrae. In a 8 mm specimen all cartilaginous elements are well developed, however, not all of the bony elements are developed, and those which have already formed, are in the initial stages of development.

In *Gallichthys* chondral stages in the tail are passed by hypurals and two terminal haemal arches, and in the anterior region of the body one-two neural arches and one haemal one. In *Callichthys*, 55 mm in length, are developed: one cartilaginous neural arch at the beginning of the trunk and two cartilaginous hypurals. In all of these elements the cartilage is very young. There are no bony arches present in this stage. In a 6.5 mm young specimen the only cartilaginous (cartilage very young) haemal trunk arch has made its appearance. Likewise, has arisen the first bone in the vertebral column: behind the cartilaginous neural arch are developed 3 pairs of bony neural arches, the bone of the latter being at the initial stage of development. In a 7.5 mm young specimen are developed the two cartilaginous (cartilage very young) haemal arches adjoining the hypurals. Thus, in this stage all cartilaginous elements of the axial skeleton are present, while not all of the bony arches are developed.

Hence, as a rule, cartilaginous arches appear in the ontogeny before the bony ones. If we take a series of consecutive neural or haemal arches proceeding from the first trunk vertebra backwards, or from the first hypural forwards and backwards, then in all the cases investigated by me the bony arches appear after the cartilaginous. Deviations from this rule can be observed but on comparison of series of arches starting to develop from different centra (i. e. from the occipital arch, on one hand, and the first hypural, on the other), or when comparing the neural arches with the haemal ones. I shall not dwell upon the importance of these deviations. To this question I expect to devote a separate paper.

The sequence in appearance of vertebral arches in different forms is not quite alike. Thus, in *Ostariophysi* possessing the Weberian apparatus, the arches of the anterior trunk region appear relatively earlier than in other fishes *. And we see that owing to this earlier appearance, the haemal arches of the trunk which are usually the first to lose their cartilage, are in *Cyprinidae* those that retain it, whereas the haemal arches of a considerable part of the tail do not pass through chondral stages. This fact is interesting because it shows with the utmost clearness what connexion there is between the omission of cartilaginous stages and the time of appearance of the element in the ontogeny.

It should be noted that the omission of cartilaginous stages in the development of arches occurs in representatives of all the suborders of *Teleostei* investigated by me. Special mention should be made of the fact that among representatives of one and the same suborder, or even within a smaller group,

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* The early appearance of arches in the anterior trunk region in *Ostariophysi* is caused according to Matveev (4), by the development of the Weberian ossicles connected with the first trunk vertebrae.

occur forms comparatively rich in cartilage, as well as those almost entirely without cartilaginous arches. In other words, the process of omission of chondral stages in the development of vertebral arches evolves parallel in different groups. Thus, among the *Siluridae*, in *Amiurus* all haemal and neural arches are cartilaginous, except the neural arches over the hypurals, whereas in *Callichthys* the cartilage vanishes almost from all the arches. Among *Acanthopterygii* only the haemal arches of the trunk and the neural ones above the hypurals do not pass through cartilaginous stages, and in *Gobius* chondral arches persist but in a few vertebrae of the terminal trunk region. In the haddock, according to Farugi [5], neither the neural nor the haemal arches pass through chondral stages, and in another representative of *Anacanthini*, the *Lota* (a 27 mm young specimen)*, I have found chondral neural arches in the trunk.

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Received
9. V. 1939.

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* I have not had any earlier stages of the caudal region of the 27 mm specimen.

Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1939. Volume XXIII, № 9

EVOLUTIONARY MORPHOLOGY

PHENOMENON OF WING REDUCTION IN SOME *HEMIPTERA*

By I. F. TRUKHANOV

(Communicated by I. I. Schmalhausen, Member of the Academy, 5. V. 1939)

Hemiptera are known to comprise quite a large number of wingless forms and forms with a varying degree of wing reduction. Within the limits of the same species occur both males and females with shortened or entirely reduced wings.

The varying degree of wing reduction is expressed in the terms of Peneau (1905) who established in regard to certain water striders (*Hydrometridae*), according to the length of the wings and that of the abdomen, six forms— Forma *macroptera*, f. *subrachyptera*, f. *brachyptera*, f. *sumicroptera*, f. *microptera*, f. *aptera*.

Kirkaldy (1899), Peneau (1905), Poisson (1921—24), Larsen (1931) who investigated wing polymorphism in certain bugs, continued their studies to wings in the imago without discussing their development. It was necessary to fill this gap, inasmuch as the reduction of organs cannot be explained without knowledge of their ontogeny. Thus, a study of the development of reduced wings in various bugs became of prime importance.

This is intended to be a very concise report of the results of a study of wing reduction in the ontogeny of the following species: fam. *Pyrrhocoridae*, *Pyrrhocoris apterus* L., forma *brachyptera*, family *Miridae*, *Miris dolobratus* L., *f. brachyptera*, fam. *Gerridae Aguarius najas* Degeer, f. *aptera*, *Ag. paludum* Fabr., f. *brachyptera*, *Limnotrochus lacustris* L., f. *brachyptera*.

Pyrrhocoris apterus L. Among individuals of this species occur males and females with reduced wings and a small percentage of full-winged forms. Reduction of the anterior wings affects the apical part (membrane), the range of fluctuations in its size being insignificant. The reduction of the posterior wings is correlative with that of the elytra.

In the course of development of wings in f. *macroptera* in the ultimate, 5th larval stage the wings are fully enclosed in the wing-cases (Fig. 1*A*). At the outset the developing elytra exhibit a rich venation (Fig. 1, *A*). They have *C*, *Sc*, *R*, *M*, *Cu* and *A* veins. Then part of the veins become reduced, and as a result just a small amount of them are left in the corium and clavus of the hemielytra of the imago (Fig. 2, *A*); thus, even in f. *macroptera* the wings in the course of their development undergo a certain reduction in regard to venation. According to D. M. Fedotov, such a phenomenon is observed in males and females of the winter-moth (*Cheimatobia brunata*) in which the veins of the imaginal wings become likewise partly reduced. Of the nervures in the posterior wings of *Pyrrhocoris apterus* L. f. *macroptera* only the marginal one undergoes partial reduction.

The f. *brachyptera* larvae in the terminal stage of development possess wing-cases similar in size to those in larvae of the same state in normal winged forms—f. *macroptera*, but there is a striking difference in the size of the wing-cases and the rudiments of the imaginal wings they are enclosing. The wings are not filling up the cases entirely, leaving some free space, owing to the reduction of the apical portion of the wing (Fig. 1,*C*). In the imaginal wings of f. *brachyptera* larvae in the terminal stage first complete venation in the regions of corum and clavus (Fig. 1, *C*) is observable, the same as in a normal wing (Fig. I, *A*). Before moulting a partial reduction of the veins in the imago takes place—in the regions of corium and clavus, and consequently the wings of the adult bug exhibit poor venation (Fig. 2, *C*). As we have seen, a similar reduction in venation occurs also in the anterior wings of

Fig. 1.

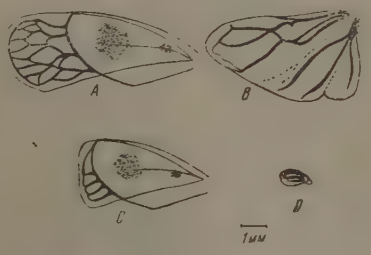

Fig. 2.

normal winged forms. Rudiments of imaginal posterior wings in f. *brachyptera* are formed in greatly reduced condition as compared to those in f. *macroptera*; reduction ensues in the first half of the 5th stage, envolving the apical part, the anterior and posterior margin of the wings. A wing thus reduced is shown in Fig. 1, *D*. The venation in the reduced posterior wings, as compared to those normally developing, is rather simplified. The same simplified nervuration is characteristic of the imaginal wings of this form (Fig. 2, *D*).

Miris dolobratus L. This species exhibits a wing dimorphism—males with normally developed wings, females short-winged, with a reduced membrane. The development of wings in females up to the last larval stage proceeds in the same manner as in males. In the terminal stage a sharp differentiation sets in—the rudiments of imaginal elytra and wings formed in the females are smaller than in the males. Like in *P. apterus* L. in f. *brachyptera* occurs a reduction of the membrane, and consequently a considerable space of the apical portion of the elytra is free from the wings. However, the veins of the elytra in the regions of corium and clavus are not undergoing reduction, neither does it take place in the development of wings in the males of this species; whereas in the course of development of posterior wings degeneration of part of the veins is observed, namely, the shortening of the subcostal and complete reduction of the cubital vein.

Aguarius paludum Fabr., f. *brachyptera*. No differences are observed in the development of wings in f. *brachyptera* and *macroptera* up to the 5th (terminal) larval stage. The elytra in the two forms are similar in length and shape. Not before the 5th stage has ensued, are the rudiments of the imaginal wings formed in greatly reduced condition taking up two thirds of the wing-case. Like in the above-described species, reduction occurs in the apical part of the anterior and posterior rudiments of the imaginal wings. In none of the stages of development is observed a reduction in the veins of the wings. Both pairs of wings in the f. *brachyptera* imago display the same nervuration as the wings in f. *macroptera*, the first-mentioned form

being distinguished by its imaginal wing rudiments forming in a reduced condition and reaching in the imago—the anterior to the 4th and the posterior to the 3rd abdominal segment.

Aguarius najas Degeer, f. *aptera*. The development of the wings in f. *aptera* up to the penultimate, i. e. the 4th larval stage, proceeds normally, practically not any different from the development of wings in f. *macroptera* and *brachyptera Ag. paludum*. Only during the 4th stage begins the reduction of the wing rudiments which continues into the ultimate, 5th stage and leads to the complete disappearance of wings in the imago of the form concerned.

Limnotrechus lacustris L., f. *brachyptera*. Up to the ultimate, 5th stage the wing development in f. *macroptera* and f. *brachyptera* proceeds on similar lines. In size they do not differ from one another. The anterior pair of imaginal wings, while developing in f. *macroptera*, is of a considerably larger size than the anterior wings in f. *brachyptera*, the reduction affecting the apical part of the wings—the membrane, i. e. reduction occurs in the same portion of the wings and in the same state (the 5th stage) as in *Ag. paludum* F. f. *brachyptera*. Once formed, the veins suffer no reduction during the entire period of wing development. The only difference between the anterior wings in f. *macroptera* and f. *brachyptera* is that of size. Rather dimunutive rudiments of the posterior imaginal wings are formed taking up but one-third of the wing-cases. The ratio of the size of the wing rudiments to that of the elytra corresponds to the ratio of the size of the wings to the elytra in *P. apterus* L., f. *brachyptera*; they resemble each other likewise in venation; namely, in the hind-wing of *Limnotrechus lacustris* L., f. *brachyptera* 3 veins are formed which persist throughout the entire course of development of wings, and are retained in the wing of the imago.

Conclusions

1. In the investigated *Hemiptera* f. *brachyptera* the wing development up to the terminal stage proceeds on similar lines with that in f. *macroptera*. Reduction occurs in the 5th stage when rudiments of the imaginal wings in f. *brachyptera* are formed in reduced condition as against the issuing normal wings.

2. In *Ag. najas* Degeer, f. *aptera* the reduction of wings starts in the 4th larval stage, continues in the 5th stage as well, and leads to the disappearance of wings in the imago.

3. There are 2 modes of wing reduction in *Hemiptera*: 1) aphanitism and 2) a mode intermediate between aphanitism and rudimentation. The former mode was found by A. N. Sewertzov as characterizing the reduction of organs in vertebrates, the latter was referred by D. M. Fedotov to invertebrates.

a) In *Pyrrhocoris apterus* L., *Miris dolobratus* L., *Ag. paludum* F. f. *brach.*, *Limnotrechus lacustris* L. f. *brach.* the wing reduction proceeds according to the intermediate mode.

b) In *Aguarius najas* Deg. f. *aptera* aphanitism applies.

4. It should be noted that the stronger reduction in *Ag. najas* Deg. is concomitant with an earlier initial reduction in the ontogeny.

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Received
5. V. 1939.

Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1939. Volume XXIII, № 9

PALEONTOLOGY

AGE OF «POLTAVA» SANDS IN THE VICINITY OF SHESTERINTSY (KIEV PROVINCE)

By N. V. PIMENOVA

(Communicated by A. A. Borissiak, Member of the Academy, 20. IV. 1939)

In the vicinity of the village Shesterintsy in the south of the Kiev province was worked a lens of brown coal, carrying fossil plants which have led to the identification of the age of this lens. And inasmuch as the brown coal layer was bedded here directly in white quartz sands of the Poltava stage, the flora determined by the author indicates at the same time the age of the Poltava sands of this region.

The stratification of this interesting bed*, in rough outline, is as follows: the Quaternary deposits of total thickness up to 30 m including brown clays and loess-like loams, are underlain by a layer of typical Poltava sand 18—22 m thick. Beneath are lying greenish sands of the Kharkov stage. Both in the white quartz and in the Kharkov sands occur seams of brown coal. The seam which was mined in the village Shesterintsy was lying directly in the white sands at a depth of 12 m below the surface, and was underlain by a 4 m thick layer of identical sand, separating it from the underlying green Kharkov sand.

The entire brown coal lens was of rather limeted size. Some of the coal sections presented a solid mass of various plant remains, mainly monocotyledones, their state of preservation, however, not allowing of identification. Other varieties had a more earthy character, and these were precisely the ones containing excellent imprints of plants bearing no traces of transfer, so that the most minute of them, like *Salvinia*, retained all the details of their structure, and occasionally large leaves were also perfectly well preserved. Evidently, we are here dealing with a small lake which became gradually filled up with remains of plants that were growing in its waters and on the shore. The composition of the flora is as follows:

1. *Osmunda* sp.
2. *Salvinia Mildeana* Goepp.
3. *Glyptostrobus europeus* Hr.
4. *Taxodium distichum miocaenum* Ung.
5. *Sequoia Sternbergii* Hr.
6. *Cannophyllum* sp.
7. *Spirematospermum Wetzleri* Chandl.
8. *Phragmites oeningensis* Hr.
9. *Populus amplifolia* n. sp.
10. *Myrica lignitum* Hr.
11. *Alnus Kefersteinii* Ung.
12. *Fagus* sp.
13. *Quercus apocynophylla* Ett.
14. *Quercus quinquedentata* n. sp.
15. *Cinnamomum Scheuchzeri* Hr.
16. *Persea* sp. n.

1) For details cf. the work by Prof. V. N. Chirvinsky and V. T. Syabry, 1935.

17. *Laurus primigenia* Ung.
18. *Laurus ocoteaefolia* Ett.
19. *Laurus princeps* Hr.
20. *Laurus Reussii* Ett.
21. *Laurus Haueri* Ett.
22. *Acer trilobatum* Al. Br.
23. *Acer crassinervium* Ett.
24. *Tetrapteris sagoriana* Ett.
25. *Grewia crenata* Hr. (?) *
26. *Tilia* sp.
27. *Cinchonidium plumeriaefolium* Ett.
28. *Diospyros brachysepala* Al. Br.
29. *Myrsine* sp. (*Endymionis* Ung.?)
30. *Apocynophyllum dubium* Staub.
31. *Nerium oleander* L.
32. *Nerium* sp.

Total—24 genera and 32 species.

On examination of this list, we find that forms such as *Osmunda*, *Glyptostrobus*, *Taxodium*, *Cannophyllum*, *Myrica*, *Laurus*, *Cinnamomum*, *Persea*, *Grewia*, *Myrsine*, *Diospyros*, *Apocynophyllum* and *Nerium*, i. e. 13 genera are mostly warmth-loving plants, partly forms of the Mediterranean, partly of the monsoon region, though the genus *Osmunda*, *Myrica* pass into the cold-temperate zone.

Salvinia, *Sequoia*, *Phragmites*, *Populus*, *Alnus*, *Fagus*, *Quercus*, *Acer*, *Tilia*—genera whose representatives were found in Shesterintsy, are largely confined to the temperate climate, although they are known to overlap their range and to penetrate into warm zones, partly forming special sections, for instance, of the genus *Quercus*.

Lastly, *Tetrapteris* and *Cinchonidium* are plants whose temperature optimum cannot be decisively settled. *Cinchonidium* occurs in Priesen in association with *Acer*, *Salvinia* and other plants identical with representatives of our flora. The same applies to *Tetrapteris* from Sagor. The flora from the latter is in general close to ours, apparently, climatic conditions prevalent at the time of the existence of these two floras were identical for Sagor and Shesterintsy.

As far as *Spirematospermum* is concerned, this plant is known from Eocene to Pliocene, i. e. has a wide climatic range. F. Kirchenheimer, having made a detailed study of the seeds of *Spirematospermum*, proves that it belongs to the family *Zingiberaceae*, i. e. to plants of the tropical zone. However, though *Spirematospermum* is known from Middle Eocene, it is still characteristic of the Miocene brown coal deposits of Germany (and according to A. Krishtofovich and P. A. Nikitin, of the Miocene of Siberia). Not only separate seeds, but whole fruits too are known. Thus, the above-mentioned plant was not restricted exclusively to the tropical climate of the Eocene, but was prospering during the temperate Miocene and Pliocene epochs.

Furthermore, on the one hand, laurel, for example, is representative not of the tropical climate alone (being widely spread among the subtropical and Mediterranean flora, and on the Black sea coast); they were found, alongside of forms of the temperate climate, in typical Miocene deposits of Bilin, Krynka, etc. On the other hand, among tropical and subtropical floras there is no such prevalence of forms with dentate leaves, like *Alnus*, *Acer*, *Tilia* and others which we have met among the Shesterintsy flora.

Thus, we have every reason to believe that the climate which prevailed at the time of the Shesterintsy brown coal formation, though comparatively warm, warmer than the present one in the given region, was still appreciably colder than during the time of the Ekaterinopol coal formation associated with palms, sequoias, etc., the Ekaterinopol bed being situated 25 km south of Shesterintsy, but in time widely remote from the latter.

Comparing the brown coal flora of Shesterintsy with West-European floras whose age is quite definitely established, we note that nearest to the latter stands the Sagor flora sharing 15 species with the one of Shesterintsy.

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* According to the latest interpretations, they, possibly, belong to the genus Trochodendron.

As to other localities, the Shesterintsy flora has 10 forms in common with both the Schichow and Kutschlin flora, 9—with the floras of Priesen, Zsilthal and Schönegg, 7—with Sused, 5—with Göhren. With others again it shares 3—2—1 forms with each. In the latter case forms common to them have a wide vertical range of distribution, like *Sequoia Sternbergii*, *Myrica lignitum*, *Laurus primigenia*, *Acer trilobatum*, *Cinnamomum Scheuchzeri*.

Among the USSR beds, Molotychi which, according to its flora, stands very close to Sused and Sagor and is referable to the Aquitanian, has very few forms in common with Shesterintsy, and the Timsk flora has none whatsoever.

Table of Synchronism of the Flora of Shesterintsy and Other Beds

Flora of the village Shesterintsy	Häring	Göhren	Sotska	Priesen	Sagor	Jesuitengraben	Steyermark	Zsilthal	Schönegg	Schichow, Kutschlin	Leoben	Radoboj	Sused	Preschen	Schossnitz	Ville	Tim	Molotychi	Sterlitamak	Niederlausitz
1. *Osmunda* sp.				×				×												
2. *Salvinia Mildeana* Goep.				×	×								×	×	×				×	
3. *Glyptostrobus europeus* Hr.		×			×			×	×	×										
4. *Sequoia Sternbergii* Hr.	×		×		×										×				×	×
5. *Taxodium distichum* Ung.		×				×		×	×											
6. *Cannophyllum* sp.									×											×
7. *Spirematospermum Wetzleri* Chandl																×			×	
8. *Phragmites oeningensis* Hr.				×	×															
9. *Populus amplifolia* sp. n.																				
10. *Myrica lignitum* Hr.		×		×	×				×							×				×
11. *Alnus Kefersteinii* Hr.				×	×	×		×	×	×			×	×					×	×
12. *Fagus* sp.																				
13. *Quercus apocynophylla* Ett.					×															
14. *Quercus quinquedentata* n. sp.																				
15. *Cinnamomum Scheuchzeri* Hr.	×		×	×	×	×		×	×	×	×	×	×							
16. *Lauris primigenia* Ung.			×		×	×		×	×	×			×					×		
17. *L. princeps* Hr.					×	×		×	×				×							
18. *L. Reussii* Ett.										×										
19. *L. Hateri* Ett.					×															
20. *L. ocoteaefolia* Ett.					×					×										
21. *Persea* sp. n. (?)																				
22. *Acer trilobatum* Al. Br.		×		×	×			×		×						×				×
23. *A. crassicervium* (?)						×	×			×			×							
24. *Tetrapteris sahoriana* Ett.					×															
25. *Grewia crenata* Hr.				×		×		×		×										
26. *Tilia* sp.																				
27. *Cinchonidium plumerifolia* Ett.				×																
28. *Diospyros brachysepala* A. Br.		×			×	×			×	×									×	
29. *Myrsine* sp. *(enoymiois ?)*					×				×			×	×							
30. *Apocynophyllum dubium*								×												
31. *Nerium oleander* L.		-																		
32. *Nerium* sp.																				
	2	5	3	9	15	8	1	9	9	10	1	1	7	2	2	1	0	1	5	5

The Sterlitamak flora recently described by A. Krishtofovich shares 5 species with the Shesterintsy flora. The floras of Sagor, Priesen and Zsilthal which share the greatest number of forms with the Shesterintsy flora, pertain to the Aquitanian, i. e. to its upper horizons, while the Schönegg, Kutschlin and Schichow floras refer already to the Langhian (Burdigalian).

Sused is attributed to the Helvetian or to the Second Mediterranean stage. In other words, all the enumerated floras refer to the close of the Oligocene and beginning of the Miocene, or in general to the base of Miocene, the Aquitanian stage being regarded as Miocene. With floras belonging to the Middle and Upper Miocene, as well as with more ancient floras (Häring) the Shesterintsy flora has very few forms in common.

However, the Shesterintsy flora has forms peculiar to higher horizons, as, for instance, *Salvinia Mildeana*, known from the 2nd Mediterranean stage, from Sarmatian deposits, from the Upper Miocene of Schossnitz etc. And *Spirematospermum*, though known from the Eocene, in association with *Acer trilobatum*, *Salvinia Mildeana* etc., resembles more the Miocene phytocoenoses of German brown coal deposits.

Thus, the brown coal flora of Shesterintsy is to be recognized as Lower Miocene owing to its resemblance to typical Lower Miocene (Aquitanian) floras.

Hence the Poltava sands in the south of the Kiev province are Lower Miocene, and by no means Oligocene in age, whereas the Kirovgrad region is typical Oligocene (Tongrian).

The character of the Shesterintsy flora differs sharply from floras of the Paleogene. While in the region of Mogilny, Bolanschina and Kirovgrad were dominant evergreen forms, like *Cinnamomum*, *Persea*, tropical species of *Quercus*, *Myrtophyllum*, along with *Palmae* and *Dryophyllum*, the Shesterintsy flora exhibits a prevalence of deciduous plants, and a decline of evergreen forms, represented here but by a negligible number of species of the most resistent evergreen types, mainly by laurel. The Shesterintsy flora is a transitional flora which had absorbed the first elements of the Turgai flora, but has not yet lost some of the «Poltava» forms (persisting, for example, in the Caucasus and farther, as *Laurus*).

This most fascinating moment of change in the character of floras is traceable in still younger floras of southern USSR entombed in white quartz sands.

We know of such young floras either fom the south, or south-east (Odessa, Kherson, Orekhov, Krynka etc.), as if the sands in the course of their gradual advancement encroach more and more upon new regions, burying in their depths more and more recent floras.

It is well nigh possible that on the territory extending between the Crystalline area and the region of the Donetz basin new floras will be discovered which will link up the floras of the Paleogene of Ukraine, through the Shesterintsy locality, with Upper Miocene floras of Krynka which were also found in the region of development of «white quartz sands in the southern USSR», however resting directly upon the sands of the Kiev stage with *Spondylus Buchii*.

Received
20. V. 1939.

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